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On the Solution of the Fokker-Planck Equation using Reduced Spectral Basis Function Method

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ABSTRACT

The Fokker-Planck equation plays an important role in physics and applied mathematics. In this study we are interested in its application to non-Newtonian flow problems.

Many natural and synthetic fluids are viscoelastic materials, in the sense that the stress endured by macroscopic fluid element depends upon the history of the deformation experienced by that element. Viscoelastic phenomena are primarily due to molecular forces which arise from the orientation of polymer chains in the liquid. Polymers immersed in a solvent can be modelled as elastic dumbbells consisting of two beads and an interconnecting spring. By starting from a coarse-grained description of these polymer chains, a partial differential equation for the probability density of the microstructural quantities can be derived. This equation is known as the Fokker-Planck equation.

For some non-Newtonian models, one can derive a macroscopic constitutive equations for the stress starting from this kinetic theory model. For example, a Hookean dumbbell model is equivalent to the Oldroyd B equation. In general, however, it is impossible to derive a closed-form constitutive equation for the stress equivalent to a kinetic theory model. This makes kinetic theory models generally more difficult in terms of numerical prediction. In this context, micro-macro methods of computational rheology that couple the coarse-grained microscopic descriptions of polymer conformations using kinetic theory to macroscopic scale continuum mechanics have an important role to play. In a micro-macro simulation, the conservation equations are solved together with a model of kinetic theory. This approach is much more demanding in computer resources than more conventional continuum simulations that integrate a constitutive equation to evaluate the contribution to the stress tensor. On the other hand, micro-macro techniques allow the direct use of the kinetic theory models and thus avoid potentially harmful closure approximations.

Mathematical models for polymer solutions based on kinetic theory can usually be described in two formally equivalent ways: via stochastic differential equations, or via deterministic Fokker-Planck equation. Different methods has been introduced for the solution of the stochastic differential equation for

the dumbbell connector vector including CONNFFESSIT, the method of Brownian configuration fields. Numerical methods based on wavelet approximations developed to solve the diffusion equations that arise from kinetic theory models of polymer dynamics for the Doi model, spectral method for the FENE dumbbells model, QMC approach, method based on using Galerkin discretisation and wavelet preconditioner for high-dimensional problems.

Also, with the help of Fokker-Planck equation one can put different effects into models, since computation provides for each polymer molecule in the motion. Furthermore, to gain a physical insight it may be necessary to consider a large number of dumbbells, and solutions to the resulting high-dimensional Fokker-Planck equation are difficult to obtain, even for the special cases mentioned above.

Numerical approximations for high dimensional problems are infeasible using standard discretisation techniques due to the excessive computational demand required. One appealing strategy that circumvents this limitation is based on the use of a reduced approximation basis within an adaptive procedure that makes use of an efficient separation of variables [1,2]. The new approximation functions are defined in the whole domain and they contain the most representative information of the solution. It also dramatically reduces the number of degrees of freedom required by the approximation. The advantage of this method is it can be adapted to any discretisation of the problem. In the work of Ammar et al. [1,2] a Galerkin variational formulation was used. The novelty of this work lies in the use of the spectral element method which has better convergence properties.

The reduced basis method with a spectral element discretisation has been tested on one and two-dimensional conduction equations and one and two-dimensional transient Fokker-Planck equations for the FENE model.

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