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A CONTACT PROBLEM INCLUDING BONE REMODELLING: NUMERICAL ANALYSIS AND COMPUTATIONAL EXPERIMENTS

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ABSTRACT

In this talk, a contact problem between an elastic body and a deformable obstacle, including the bone remodeling process, is numerically studied. This bone remodeling model, derived by Cowin and Hegedus ([1]), is a generalization of the nonlinear elasticity, and it is based on the fact that the living bone is continuously adapting itself to external stimuli. Since this process has an enormous effect on the overall behaviour and health of the entire body, the ability of these models to predict the bone remodeling is of great importance.

During the last ten years, some papers dealt with mathematical issues of these models as the existence and uniqueness of weak solutions under some quite strong assumptions (see, e.g., [4] or [5]) or the analysis of an asymptotic one-dimensional contact problem ([2]). Here, the contact is modelled using the normal compliance contact condition which can be found, for instance, in [3], and the full mechanical problem is considered.

The variational problem is written as a nonlinear variational equation for the displacement field, coupled with a first-order ordinary differential equation to describe the physiological process of bone remodeling. An existence and uniqueness result of weak solutions is stated, which is proved following [2] by using Schauder's fixed point theorem, the Cauchy-Lipschitz-Picard theorem, the Lax-Milgram lemma and regularity results.

Then, fully discrete approximations are introduced based on the finite element method to approximate the spatial variable and an Euler scheme to discretize the time derivatives. A main error estimates result is obtained, from which the linear convergence of the algorithm is derived under suitable regularity conditions on the continuous solution. Finally, some numerical results, in one, two and three dimensions, are presented in order to demonstrate the performance and the accuracy of the algorithm. As an example, in Figure 1 we provide the results obtained in a two-dimensional example, for a compression force linearly increasing through the horizontal direction but time-independent, and applied on the upper horizontal boundary. As can be seen, the displacements decrease due to the bone remodeling.

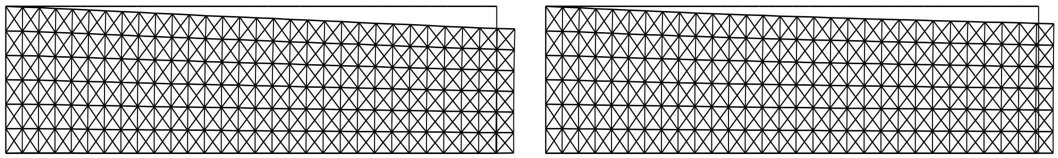

Figure 1: Reference configuration and displacement field at initial time (left) and after 105 days (right).

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