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Shape Sensitivity Analysis of Elastic Thin Shells

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ABSTRACT

This study concerns the application of shape sensitivity analysis to thin shells within the framework of linear elastic theory, Koiter (1970) [1]. The basic idea behind almost all theories of shells is to reduce the analysis over the middle surface by means of simplified kinematical assumptions. According to this, the shape of a shell may be characterized by the geometry of the corresponding middle surface.

The shape change concept is furnished making use of an analogy with continuum mechanics. More specifically an analogy between shape change and the motion of a body from the initial configuration to a deformed configuration. In the particular case of a shell the motion of a surface embedded in the three dimensional Euclidean space is considered.

Following this methodology the attention on the shape change descriptions over the referential, initial and deformed configurations and the relations among them is focused in this work. The referential description of the shape change is furnished through a smooth mapping over a two dimensional domain and the shape derivatives are performed by the differentiation with respect to the mapping. Thus, the change of the shape can be seen as the change of the mapping, Bernadou *et al.* (1991) [2]. When the analysis is expressed over de initial middle surface an analogy can be drawn between shape change and the Lagrange approach of solid mechanics. In this case the shape derivatives are computed using the concept of material derivative. When the deformed or spatial description is adopted an analogy can be made with the Euler approach of fluid mechanics and, in this case, the shape derivatives are deduced applying the concept of the material or total (time) derivatives of spatial fields. In the last two cases, the actual and spatial descriptions, the intrinsic base, given by the unit normal vector and the tangential plane at each point of the middle surface of the shell, is used throughout this paper, Taroco and Feijóo (2004) [3].

With the aim of making a clearer presentation and, at the same time, to put in evidence the simplicity of the adopted approach, this work is limited to carrying out exclusively the shape sensitivity analysis of the total potential energy stored in a shell submitted to static loads along its boundary. Moreover, the variational formulation of the mechanical model linked to the direct method of shape sensitivity analysis lead to a very useful expression for the shape sensitivity of the total potential energy. Finally, following the above approach, a general expression for the shape derivative is obtained in terms of the strain-stress state. This expression also results in an explicit function of the tangent and normal components of the velocity field which characterizes the shape change of the shell.

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