8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

An IRBFN Cartesian Grid Method Based on Displacement-Stress Formulation for 2D Elasticity Problems

P. Le[1], N. Mai-Duy[1], T. Tran-Cong[1]* and G. Baker[2]

[1] CESRC, University of Southern Queensland, Toowoomba, QLD 4350, Australia.

[2] DVC(S), University of Southern Queensland, Toowoomba, QLD 4350, Australia.

Key Words: *RBF, collocation method, elasticity, cartesian grid, mixed formulation.*

ABSTRACT

Traditional finite element methods (FEM) and boundary element methods (BEM) have been based on weak-form formulations. Recently, weak-form meshless (meshfree) methods are being developed as an alternative approach. Weak-form methods have the following advantages [1]. a) They have good stability and reasonable accuracy for many problems. b) The traction (derivative or Neumann) boundary conditions can be naturally and conveniently incorporated into the same weak-form equation. However, elements have to be used for the integration of a weak form over the global problem domain and the numerical integration is still computationally expensive for these weak-form methods. On the other hand, collocation methods are based on strong-form governing equations and have been found to possess the following attractive advantages [1]. a) There is no need for numerical integration of the governing equations. b) The implementation is simple. However, the strong-form approach is less stable due to the pointwise nature of error minimisation. Furthermore, strong-form methods such as finite difference and pseudo spectral methods are restricted to regular domains.

Following a strong-form approach, this paper describes a new efficient method using integrated radial basis function network (IRBFN) and cartesian grid [2] for the numerical modelling of 2D elasticity problems in both regular and irregular domains. Clearly, the generation of a cartesian-grid is a straightforward task and therefore the cost associated with spatial discretisation is greatly reduced in comparison with that associated with FE generation. However, there are challenges in the handling of an irregular boundary [3]. To meet the challenges faced by the collocation methods, firstly we introduce a new approach based on displacement-stress formulation where both displacements and stresses are considered as primary variables. As a result, mixed boundary conditions are easily and directly accommodated. Secondly, a new technique based on 1D-IRBFN is introduced to accurately interpolate variables along grid lines. The strength of this technique is that irregular boundaries can be easily and accurately represented.

The new approach is illustrated with the analysis of Timoshenko beam (Figures 1 and 2) and an infinite plate with a circular cutout (Figures 3 and 4). Numerical results show that the present method achieves very good accuracy and high convergence rates for both compressible and incompressible solids and no volumetric locking effects are observed for the latter case.

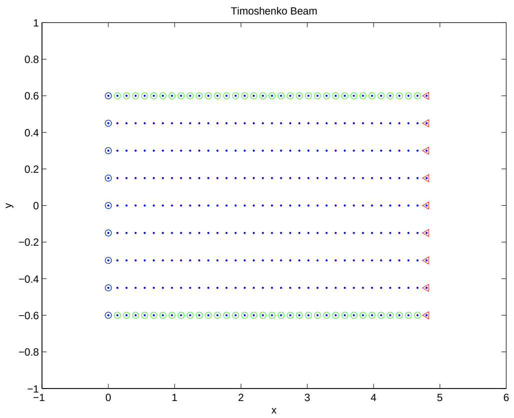

Figure 1: Timoshenko beam: domain discretization with and cartesian-grid.

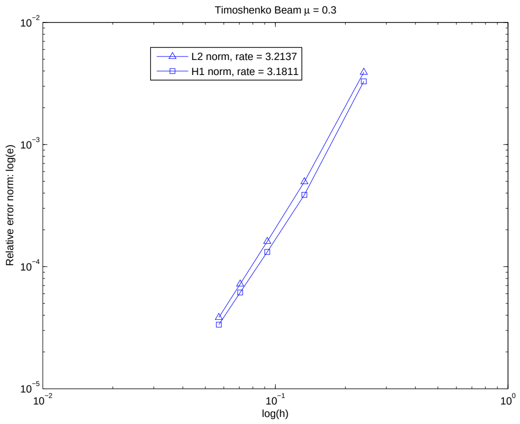

Figure 2: Timoshenko Beam problem (Poisson ratio $\mu = 0.3$): relative error norms L2 (displacement) and H1 (stress).

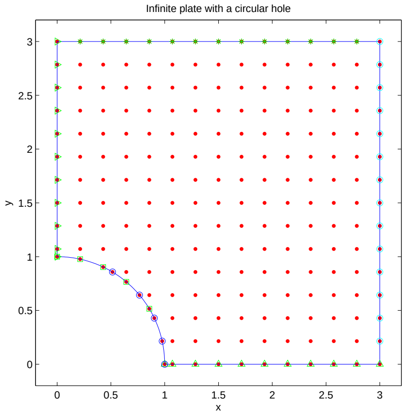

Figure 3: Infinite plate with a circular hole problem: computational domain, discretization with cartesian-grid.

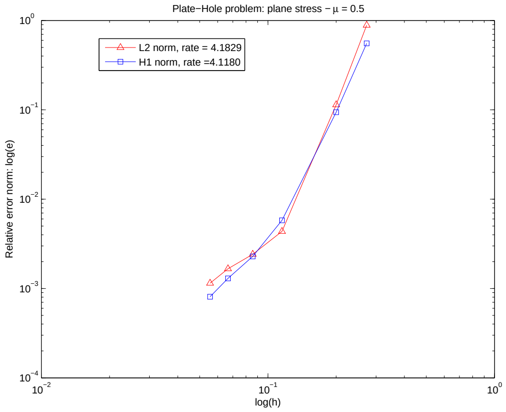

Figure 4: Infinite plate with a circular hole problem (Poisson ratio $\mu = 0.5$ incompressible material): relative error norms L2 (displacement) and H1 (stress).

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