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PARALLEL PROCESSING APPLIED TO AN UPPER BOUND FORMULATION FOR 3D LIMIT ANALYSIS COMPUTATIONS

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ABSTRACT

The kinematical theorem of Limit Analysis allows the computation of upper bounds for the collapse loads of mechanical structures presenting a perfect plastic behavior, under the assumption of non-cyclic loading. Numerical formulations derived within this framework often represent an effective tool and a competitive alternative to a plastic incremental analysis.

The objective of this work is to present a parallel processing implementation of a mixed finite element model developed for upper bound limit analysis computations.

Nowadays, and in spite of the remarkable evolution of computers performance, the determination of accurate collapse load estimates for 3D problems (the focus of this work) can still represent a significant computational effort. For this reason, we feel that resorting to parallel computing on this specific field of the computational mechanics represents a step forward

The finite element formulation described in this work is derived with the objective of complying strictly with the principals of the kinematical theorem and is, therefore, able to produce rigorous upper bounds.

The idea sustaining the present model is a simple one. When formulating the problem of limit analysis as an optimization problem,

$$\begin{aligned} \text{minimize} \quad & \mathcal{D}(\dot{\varepsilon}) - \widetilde{W}(\dot{u}) && \text{(1a)} \\ \text{subject to} \quad & \dot{\varepsilon} = B\dot{u} && \text{(1b)} \\ & W(\dot{u}) = 1 && \text{(1c)} \\ & \dot{\varepsilon} \in \mathcal{C}_c && \text{(1d)} \end{aligned}$$

it can be observed that the nonlinearity of the problem is always inherent to terms associated with the strain rate field, $\dot{\varepsilon}$, specifically the plastic dissipation rate $\mathcal{D}$ and the normality rule expressed by eq. (1d). The remaining terms, involving the velocity field, are linear ones, namely the external work rate of constant loads, $\widetilde{W}$, and variable loads, W, and the compatibility condition (1b). Based on the above

observation we devise a mixed finite element model using two independent approximations for the velocity and for the strain rate field.

Moreover, the augmented lagrangian method is used, and resorting to the Lagrangian multipliers λ_1 and λ, constraints (1b, 1c) are introduced in the objective function,

$$\min_{\dot{u},\dot{\varepsilon}} \max_{\lambda_1,\lambda} \quad \mathcal{L}(\dot{u},\dot{\varepsilon},\lambda_1,\lambda) = \mathcal{D}(\dot{\varepsilon}) - \widetilde{W}(\dot{u}) + \lambda_1(1-W) + \lambda : (B\dot{u}-\dot{\varepsilon}) + \frac{r}{2}|B\dot{u}-\dot{\varepsilon}|^2 \tag{2a}$$

$$\text{subject to} \quad \dot{\varepsilon} \in \mathcal{C}_c \tag{2b}$$

The additional penalty term, controlled by the penalty parameter r, is introduced to enforce the compatibility between the velocity and the strain rate fields. The outcome is a saddle point problem, with explicit constraints affecting the strain rate field only. The strategy adopted to solve problem (2) is based on the Uzawa algorithm using a two step relaxation scheme[1], summarized bellow:

1. Arbitrarily initialize λ, $\dot{\varepsilon}$
2. For $m \geqslant 1$
   - 2.1 Relaxation STEP 1: min $\mathcal{L}(\dot{u})$
   - 2.2 Relaxation STEP 2: min $\mathcal{L}(\dot{\varepsilon})$ $\quad \forall \dot{\varepsilon} \in \mathcal{C}_c$
3. Update λ
4. If convergence not reached GOTO 2

Of note is the fact that, apart from STEP 1, all required in Uzawa algorithm can be performed independently for each finite element. In addition, STEP 1 leads to a linear system of equations. Conveniently, the governing system matrix remains unaltered during the entire iterative process; all the modifications affect only the right hand side term. Therefore, a single matrix factorization procedure is needed throughout the whole iterative process.

These characteristics make it very suitable and efficient to implement Uzawa's algorithm in a parallel processing manner. In fact, the only complex procedure required to parallelize the algorithm is the implementation of a direct parallel solver. A more detailed explanation of the method can be found in Refs.[3, 2]

REFERENCES

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