8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

OPTIMAL CONTROL OF EUTROPHICATION PROCESSES

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Key Words: *Bioreactors, Eutrophication, Optimal control.*

ABSTRACT

The eutrophication is classically defined as "the process of nutrient enrichment (usually by nitrogen and/or phosphorus) in aquatic ecosystems such that the productivity of the system ceases to be limited by the availability of nutrients. It occurs naturally over geological time, but may be accelerated by human activities (e.g. sewage or land drainage)" (Oxford English Dictionary). The effects of hypernutrification can give rise to eutrophication (the presence of nuisance or toxic algal blooms as a result of nutrient enrichment), depending upon the environmental conditions. Eutrophication of aquatic media has been considered one of the major threats to the health of ecosystems since the mid 20th Century, and the different processes and effects of eutrophication are being widely studied and documented.

Eutrophication usually occurs due to a high number of nutrient sources resulting from human activity, the main ones could be (i) the point sources (such as sewage treatment outfall pipes and storm overflows) which may be connected to a sewage pipe system, (ii) the application of commercial fertilizer and subsequent catchment run-off resulting in large quantities of nutrients to both surface and groundwater, or (iii) the animal waste which contributes significantly to nitrogen run-off, especially in rural areas. Under natural conditions, the addition of nutrients to water bodies (which stimulate algal growth) is usually a slow process that results in healthy and productive ecosystems. Nevertheless, accelerated nutrient input to ecosystems can cause excessive growth of algae leading to the degradation of environmental conditions.

Models governing eutrophication processes are based on systems of nonlinear parabolic partial differential equations with a great complexity, due to the multiplicity of phenomena appearing on them. In this work we have considered a relatively simple model where a complete set of five species is analyzed: nutrient, phytoplankton, zooplankton, organic detritus and dissolved oxygen [1]. For this complete model, different results of existence-uniqueness-regularity have been obtained by the authors.

The fundamental idea of a bioreactor consists of holding up water (rich, for instance, in nitrogen) in large tanks where we add a certain quantity of phytoplankton, that we let grow in order to absorb nitrogen from water. In the particular problem analyzed in this work we consider only two large shallow tanks with the same capacities. Water rich in nitrogen will fill the first tank, where we will add a quantity ρ_1 of phytoplankton - which we will let freely grow - to drop nitrogen level up to a desired threshold.

Inside this first tank we are also interested in obtaining a certain quantity of organic detritus - very estimated as agricultural fertilizer. Once reached the desired levels of nitrogen and organic detritus, we will filter water and pass it to the second tank, where the same operation is repeated, by adding a new quantity ρ_2 of phytoplankton. Water leaving this second fermentation tank will be usually poor in nitrogen, but rich in detritus (recovered from a second filtering) and phytoplankton. At this point, we are also interested - for economical reasons - in minimizing this final quantity of phytoplankton. Thus, the optimal control problem consists of finding the quantities (ρ_1, ρ_2) of phytoplankton that we must add to each one of both tanks, so that the nitrogen levels be lower that maximum thresholds and detritus levels be higher that minimum thresholds, and in such a way that the final phytoplankton concentration be as reduced as possible.

From a mathematical viewpoint, this problem can be formulated as an optimal control problem with state constraints, where the control (ρ_1, ρ_2) is the quantity of phytoplankton added at each tank, the state variables are the concentrations of nutrient, phytoplankton, zooplankton, organic detritus and dissolved oxygen, the objective function to be minimized is the phytoplankton concentration of water leaving the second tank, and the state constraints stand for the thresholds required for the nitrogen and detritus concentrations in each one of the tanks.

We can prove that this optimal control problem admits a (non necessarily unique) solution, which can be characterized by a first order optimality condition, involving an adjoint system to be adequately defined. In order to obtain a numerical solution of the optimal control problem we solve the state system with a finite element discretization combined with a fixed point algorithm for solving its nonlinearities (implemented in our own Fortran code), and, for minimizing the objective function, we have used the IPOPT code, an implementation of an interior point algorithm whose details can be seen, for instance, in [2]. Finally, we present several numerical results for our real-world problem.

Acknowledgement: This work was supported by Project MTM2006-01177 of Ministerio de Educación y Ciencia (Spain).

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