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A point-collocation method based on integrated Chebyshev polynomials for elliptic differential equations in irregular domains

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ABSTRACT

This paper describes a numerical approach for elliptic partial differential equations based on integrated Chebyshev polynomials and point collocation. In this approach, the starting points of the approximation process are the highest derivatives of the field variables in the given partial differential equations. Lower derivatives, and eventually the variables themselves, are symbolically obtained by integration, giving rise to integration constants that serve as additional expansion coefficients, and therefore facilitate the employment of some extra equations. It is shown that this feature provides an effective way to handle the description of non-rectangular boundaries in Cartesian grids. As a result, there is no need to transform an irregular domain into a regular one and the governing equations remain in the simple Cartesian form. In addition, the use of integration also improves the quality of the approximation of derivative functions owing to its smoothness and stability.

Numerical studies are conducted for Poisson equation on a circular domain (Figure 1) with the following driving and exact functions

$$b(x,y) = -2\sin(\pi x)\sin(\pi y), \tag{1}$$

$$u_e(x,y) = \frac{1}{\pi^2}\sin(\pi x)\sin(\pi y). \tag{2}$$

Relative L_2 errors of the solution u are shown in Table 1. Results indicate that the technique preserves an exponential rate of convergence with grid refinement.

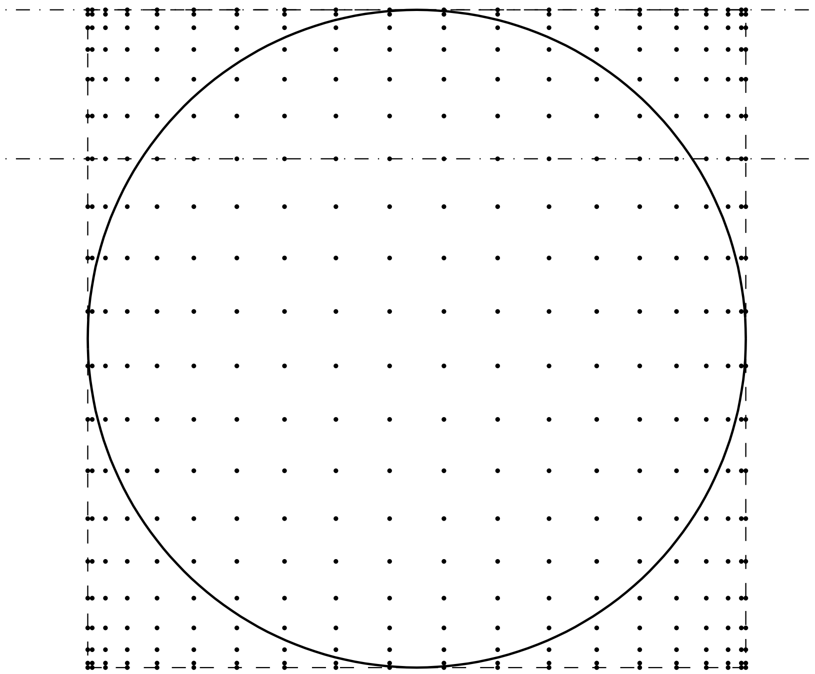

Figure 1: Domain discretization for integrated Chebyshev approach.

Table 1: Poisson equation on a non-rectangular domain: relative L_2 errors (N_e) by the integrated Chebyshev approach

$n_x \times n_y$	$N_e(u)$
6×6	3.58×10^{-3}
8×8	8.08×10^{-5}
10×10	6.36×10^{-7}
12×12	7.27×10^{-9}
14×14	4.49×10^{-11}
16×16	3.21×10^{-13}
18×18	4.85×10^{-14}
20×20	3.26×10^{-14}