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A multidomain integrated RBF collocation method for elliptic differential equations.

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ABSTRACT

This paper describes a non-overlapping multidomain radial-basis-function (RBF) technique that improves smoothness of the approximate solution across subdomain interfaces. In general, the procedure is similar to the Schur complement approach where the interface solutions are obtained before the subdomain solutions. In particular, subdomain problems are solved using the integrated RBF collocation technique. On each subdomain, highest-order derivatives in the given differential equation (DE) are approximated using RBF networks, and lower-order derivatives and the variable itself are then obtained through integration. The present integrated RBF technique introduces integration constants which provide an effective means to collocate the DE on the interfaces. Continuity enforcements employed in the Schur complement system and the satisfaction of the DE on the subdomain interfaces in the subdomain discrete system leads to a solution that is continuous across the interfaces with one order higher than that achieved with conventional domain decomposition techniques.

The attractiveness of the present higher-order domain decomposition technique is demonstrated through the solution of one-dimensional elliptic problem (Table 1) and two-dimensional elliptic problem (Table 2).

Table 1: $d^2u/dx^2 = -\sin(\pi x)$, $-1 \leq x \leq 1$, Dirichlet boundary conditions, exact solution $u_e = \sin(\pi x)/\pi^2$, 5 subdomains: Relative L_2 errors of u computed at a test set of 201 uniformly distributed points by the multidomain differentiated and integrated RBF techniques. The latter outperforms the former regarding both accuracy and convergence rate.

n	$N_e(u)$	
(Points/subdomain)	Differential approach	Integral approach
11	4.34×10^{-1}	4.92×10^{-4}
21	4.25×10^{-1}	1.23×10^{-4}
31	4.22×10^{-1}	5.53×10^{-5}
41	4.21×10^{-1}	3.12×10^{-5}
51	4.20×10^{-1}	2.00×10^{-5}
61	4.20×10^{-1}	1.39×10^{-5}
71	4.20×10^{-1}	1.02×10^{-5}
81	4.19×10^{-1}	7.83×10^{-6}
91	4.19×10^{-1}	6.19×10^{-6}
101	4.19×10^{-1}	5.02×10^{-6}

Table 2: Poisson equation, $-1 \leq x, y \leq 1$, Dirichlet boundary conditions, exact solution $u_e(x, y) = \sin(\pi x)\sinh(y) + \cosh(2x)\cos(2\pi y)$: Relative L_2 errors and their orders by the one-domain and multidomain integrated RBF methods. The latter is able to employ much larger numbers of collocation points (e.g., up to 40,400 grid points used here). It is remarkable that there is only a slight reduction in convergence rate from the case of single domain to the case of 16 subdomains.

Single domain		Sixteen subdomains	
$n_x \times n_y$	$N_e(u)$	$n_x \times n_y$/subdomain	$N_e(u)$
3×3	1.0275e+0	3×3	9.0074e-2
7×7	2.6783e-2	7×7	2.6275e-3
11×11	4.1082e-3	11×11	5.7330e-4
15×15	1.1806e-3	15×15	2.0881e-4
19×19	4.6597e-4	19×19	9.7596e-5
23×23	2.2276e-4	23×23	5.2946e-5
27×27	1.2160e-4	27×27	3.1731e-5
31×31	7.3428e-5	31×31	2.0427e-5
35×35	4.8094e-5	35×35	1.3881e-5
39×39	3.3691e-5	39×39	9.8421e-6
43×43	2.4953e-5	43×43	7.2226e-6
47×47	1.9339e-5	47×47	5.4544e-6
51×51	1.5545e-5	51×51	4.2197e-6
	$O(h^{3.51})$		$O(h^{3.07})$