

A MULTISCALE FAILURE FRAMEWORK FOR THIN HETEROGENEOUS STRUCTURES

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ABSTRACT

Computational analysis of failure in heterogeneous materials when subjected to dynamic loads such as impact, blast and crushing has proven to be a significant challenge. On one hand, characterization of the heterogeneous deformation patterns, failure processes such as delaminations, interfacial debonding, microcrack initiation and propagation requires detailed modeling in the microstructural level. On the other, limitations on computational capabilities restricts full resolution of small scales across macroscale structures. There is a clear need of adaptive computational tools capable of efficiently evaluating the failure response of large scale structural systems, while accounting for the salient features of the deformation and failure processes occurring simultaneously in the micro- and macro-scales.

Oskay and Fish [1] recently introduced a computational homogenization framework named *Eigendeformation based homogenization (EHM) method* for failure and fragmentation of heterogeneous materials. This framework is a generalization of the mathematical homogenization theory with eigenstrains, introduced by Dvorak [2]. Particularly, the EHM approach: (a) accounts for interface failure in addition to failure of microconstituents; interface failure is modeled using so-called eigendisplacements - a concept similar to eigenstrains used for modeling of inelastic deformation of phases; eigenstrains and eigendisplacements are collectively called eigendeformation, and; (b) is equipped with an adaptive model improvement capability; it incorporates a hierarchical sequence of computational homogenization models where the most inexpensive member of the sequence represents simultaneous failure of each microphase (inclusion, matrix and interface), whereas the most comprehensive model of the hierarchical sequence coincides with a direct homogenization approach.

In the present work, we extend the ideas presented in [1] to analysis of thin heterogeneous structures using plate and shell theories. The continuum solid modeling approaches are not desirable for analysis of thin structures in two major respects: (1) The continuum models are inefficient due to increased

discretization requirements in the thickness direction. This issue is more pronounced in explicit dynamic analysis, where allowable time step size scales with the smallest element sizes in the thickness direction. (2) the continuum homogenization approaches are known to be inaccurate in the vicinity of boundaries since the condition of periodicity of the response fields - a key assumption in mathematical homogenization theory is violated.

We propose a new EHM based multiscale failure modeling framework for the analysis of heterogeneous thin structures subjected to dynamic as well as static loads. The proposed approach is a generalization of structural homogenization theory for heterogeneous plates [3,4]. In the present approach, the response is characterized by the presence of three scales: (1) scale of the structural component (macroscale); (2) scale of the periodic in-plane heterogeneities, and; (3) scale of the plate thickness. The presence of the microscopic scales (2) and (3) are represented by a three-scale decomposition of the original coordinate vector. Asymptotic analysis of the original nonlinear boundary value problem of 3-D continuum leads to a fully coupled system of 3-D microscopic continuum problems and a 2-D macroscopic structural problem. The coupling is due to the presence of inelastic processes. The inelastic processes are defined at the smallest scales in terms of the degradation of the material properties as a consequence of microcrack formation and coalescence as well as debonding at the microconstituent interfaces. The evaluation of the coupled system of micro- and macroscale boundary value problems is conducted by invoking the eigendeformation concept, in which, the inelastic fields are represented by the influence functions, which reflect the effects of microstructural heterogeneities on the macroscopic scales. They are evaluated as discrete approximations to the first and second order fundamental RVE problems. By this approach, the microscopic equilibrium is satisfied irrespective of the state of the inelastic processes and the requirement of evaluation of nonlinear microscale boundary value problems is avoided without loss of generality.

The present approach is particularly attractive since no restrictions are imposed on the microstructural geometry. Composite structural systems with complex composite microstructures (e.g., angle-interlock, z-pin) in addition to traditional laminated systems may be evaluated using the proposed framework. We concentrate our efforts on a generalization of the homogenization framework for higher order structural theories and investigate the effects of higher orders terms on the accuracy, particularly for moderately thick heterogeneous systems. Dynamic asymptotic analysis suggests that the inertial response is characterized by the presence of multiple temporal scales. Specifically, the transient motion in the thickness direction is dictated by much slower time scales compared to the in-plane directions. The presence of slow temporal scales may also be used to eliminate secular terms, preventing unbounded amplification in transient dynamics [5].

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