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A NEW DEVELOPMENT PLATFORM FOR PARAMETER CONTINUATION AND BIFURCATION ANALYSIS IN NONLINEAR DYNAMICAL SYSTEMS

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ABSTRACT

A combination of theoretical and computational tools for bifurcation analysis of dynamical systems offers distinct advantages to brute force forward time simulation [9]. Such a combination enables pre diction of behavior and response without the need for a vast collection of simulations based at distinct initial conditions. More importantly, it may offer an understanding of, and an underlying explanation for, changes in behavior and response that are not available through simple simulation.

A comprehensive bifurcation analysis of a dynamical system seeks to establish the existence of charac teristic classes of responses, such as equilibria or periodic responses. In each case, this involves locating and tracking families of such responses under variations in system parameters in a process known as *continuation* [1,5,8,10]. The study of the robustness of particular system responses further emphasizes parameter values where such families merge or terminate or where the stability characteristics of the corresponding responses change.

A number of computational tools are available for bifurcation analysis of characteristic classes of re sponse, such as equilibria, periodic trajectories, homo or heteroclinic trajectories between equilib ria and/or periodic trajectories, quasiperiodic trajectories on invariant tori, and trajectories on asso ciated stable and unstable manifolds. These include general algebraic and two point boundary value solvers for ordinary differential equations, such as AUTO (and specialized drivers, such as HOMCONT [2], SLIDECONT [3], and TC HAT [13]), MATCONT [4], and SYMPERCON [14]; boundary value solvers for delay differential equations, such as DDE BIFTOOL [6] and PDDE CONT [12]; tools for large scale systems, such as LOCA [11]; and implementations in MATLAB [7].

This presentation outlines a recent software development effort in MATLAB of a set of core toolboxes for parameter continuation of algebraic and multi point boundary value problems referred to collectively as COCO. In contrast with the packages referenced above, COCO has been formulated with emphasis on full transparency, modularity, and great generality giving developers of bifurcation packages that rely on this platform great flexibility and full access to the entirety of continuation data. In addition, COCO makes extensive use of vectorization and bandwidth reduction for sparse matrix operations to guaran tee an optimal execution time at or close to what can be expected of interpreted code. Finally, COCO

includes a unique implementation of user defined functions that allows the continuation of solutions with nontrivial properties, for example, periodic orbits with a given stability margin. The presentation discusses the philosophy behind the software development, the strategy employed in its implementa tion, and a number of model examples, including continuation of periodic orbits in hybrid dynamical systems.

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