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ON MODELING AND DISCRETIZATION ERRORS IN MULTISCALE PROBLEMS: A TWO-SCALE ADAPTIVE FE-DISCRETIZATION OF A HETEROGENEOUS STRUCTURE

*** Fredrik Larsson[1] and Kenneth Runesson[2]**

[1]Dept. Applied Mechanics
Chalmers University of Tech.
fredrik.larsson@chalmers.se

[2]kenneth.runesson@chalmers.se

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ABSTRACT

A classical approach to account for the effect of the material substructure in continuum modeling of solids involves the analytical or computational homogenization on a Representative Volume Element (RVE). This notion presumes scale separation in the sense that the subscale solutions interact only via their homogenized results on the macroscale, typically via equilibrium of suitably defined macroscale stresses (in the case of quasi-statics); hence, neither kinematic nor dynamic compatibility between the RVE's is ensured.

In the simplest (and most common) approach of "first order homogenization", it is only the macroscale deformation gradient that is used as "loading" for the subscale modeling. In this case the subscale modeling (e.g. size, boundary conditions and finite element discretization of the RVE) represent model errors in the corresponding macroscale constitutive relations. The other extreme model assumption is to completely avoid scale separation, whereby the subscale is entirely inherent in the macroscale computation, and the resulting error is that of the macroscale (finite element) discretization only.

In this presentation, we adopt a novel algorithm bridging the two extremes presented above. A generalized variational framework, denoted the Scalebridging Variational Multiscale Strategy (SVMS), is presented. This strategy incorporates, as special cases, the Variational Multiscale Method by Hughes et al. [1] as well as classical homogenization based on complete scale separation. Most importantly, the approach to the subscale modeling can be chosen adaptively based on the relation of the macro-scale mesh diameter to the typical length scale of the subscale structure. In summary, we may resort to classical homogenization in the case that the macroscale discretization is much coarser than the subscale structure, whereas no subscale modeling is needed at all in the case that the "macro-scale" discretization resolves the subscale features.

In order to benefit from the possibilities of the method, we construct an adaptive scheme based on the pertinent a posteriori error estimator. Clearly, the strategy introduces discretization errors on two scales. In this presentation, we present an error estimation technique that accounts for both these sources of errors. In this context, the sub-scale discretization error appears as a model error on the macro-scale. Following the ideas in Larsson and Runesson [2], we adopt an adaptive algorithm balancing the discretization errors on the two scales. In particular, the existing method is extended to the SVMS

whereby the subscale discretization errors only occur in regions of scale separation where some sort of subscale modeling is needed.

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