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The Robin-Robin/GMRES algorithm for fluid-structure interaction: Large added-mass effect and balloon-type problems.

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ABSTRACT

The most popular approach to fluid-structure simulation is to use domain decomposition techniques. These techniques allow to decouple fluid and structure evaluations by introducing coupling iterations. The splitting of the coupled fluid-structure system into fluid and structure sub-problems permits to reuse existing codes that have been developed for every field separately. In order to couple these two sub-problems, only interface information must be transferred between codes. This modular treatment of fluid and structure problems makes the use of these partitioned procedures very useful. Depending on the fluid-structure interface boundary conditions (transmission conditions) that are used for every sub-problem, we get different algorithms. For instance, the Dirichlet-Neumann (DN) algorithm supplements the fluid sub-problem with Dirichlet boundary conditions (continuity of velocities) and the structural problem with Neumann boundary conditions (continuity of stresses). Other alternatives are the Neumann-Neumann and Neumann-Dirichlet algorithms. These algorithms were initially used for aeroelastic applications, where the structure density is much larger than the fluid density. In those applications, the convergence of these methods is good. In fact, in some aeroelastic applications, usually involving compressible flows, the coupling is considered in an explicit way. Unfortunately, this is not always the case. The fluid acts over the structure as an extra mass localized on the interface. The importance of the so-called added-mass in the structure problem increases with the ratio ρ_s/ρ_f, where ρ_s and ρ_f are the structure and fluid density, respectively. The effect of the added-mass on the convergence properties of the previous algorithms is dramatic. When the fluid and structure densities are of the same order, like in hemodynamics applications, the added-mass effect is much more important, and the convergence of these methods deteriorates in such a way that can only be attained by over-relaxation, making the iterative process extremely slow.

In order to get partitioned procedures that keep modularity and exhibit better convergence properties, different strategies have been pursued. Initially, the Dirichlet-Neumann algorithm was used together

with Richardson iterations. In order to improve the convergence process, more elaborated Krylov techniques (the GMRES algorithm) have been suggested (see e.g. [3,2]). GMRES requires to understand the Dirichlet-Neumann method as a preconditioner for the interface equation, as usual in domain decomposition. Then, the GMRES iterative solver is used over the preconditioned interface problem. This approach is much more robust, because convergence is assured, even though for a too large number of iterations. In any case, the DN preconditioner with GMRES iterations (DN-GMRES) is also severely affected by the added-mass effect and becomes fairly expensive for applications where the added-mass effect is large. Another alternative is to consider different transmission conditions. In [1] the authors have suggested the use of Robin transmission conditions, motivating the Robin-Neumann (RN, Robin for the fluid and Neumann for the structure) and Robin-Robin (RR) algorithms, supplemented with Richardson iterations. These algorithms have much better convergence properties than DN for challenging hemodynamics applications. The choice of the combination parameters used for the obtention of the Robin interface conditions (from Dirichlet and Neumann interface conditions) is one of the key aspects of the algorithm (see [1]).

In this work, we consider the combination of the two strategies considered above: better iterative algorithms and better transmission conditions. The goal is to design RR algorithms together with a GMRES iterative solver. The first task is to write the RR algorithm as an interface preconditioner. This process not only motivates a sequential algorithm (like the one in [1]) but also new parallel algorithms. The solution of the interface problem by the GMRES method preconditioned with RR (RR-GMRES) is and analyzed using numerical experimentation. The use of GMRES coupling iterations greatly improves the convergence properties of the RR (and RN) algorithms with respect to Richardson iterations. Moreover, the RR-GMRES seems to be fairly insensitive to the parameters used in the Robin boundary conditions and it can be generalized to a wide range of applications without requiring a detailed evaluation of these coefficients, making the applicability of the algorithms much easier. In any case, RR algorithms are superior to the classical DN method, both when using Richardson and GMRES iterations.

Another challenging situation in fluid-structure interaction is the numerical simulation of problems where an incompressible fluid is enclosed in a structure or it has a free boundary where Dirichlet conditions are imposed (balloon-type problems). In this case, the use of DN leads on the one hand to a fluid sub-problem with pressure defined up to a constant, and on the other hand, to a structure sub-problem with constrained interface displacement to satisfy the mass conservation principle. Its practical implementation would require either the use of a Lagrange multiplier to enforce the structure displacement constraint (destroying the positive-definite nature of the structural problem) or the introduction of pseudo-compressibility (that requires an external loop that makes the approach extremely expensive) (see [4]). We show that RN and RR naturally overcome such difficulty because the fluid problem is not confined anymore. In particular, for those applications RR-GMRES is much better than RR-Richardson.

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