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# STATISTICAL MECHANICS OF THE FLUCTUATING LATTICE BOLTZMANN EQUATION

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## ABSTRACT

We propose a new formulation of the fluctuating lattice Boltzmann equation that is consistent with both equilibrium statististical mechanics and fluctuating hydrodynamics. The formalism is based on a generalized lattice-gas model, with each velocity direction occupied by many particles. We show that the most probable state of this model corresponds to the usual equilibrium distribution of the lattice Boltzmann equation. Thermal fluctuations about this equilibrium are controlled by the mean number of particles at a lattice site. Stochastic collision rules are described by a Monte Carlo process satisfying detailed balance. This allows for a straightforward derivation of discrete Langevin equations for the fluctuating modes. It is shown that all non-conserved modes should be thermalized, as first pointed out by Adhikari *et al.*; any other choice violates the condition of detailed balance. A Chapman–Enskog analysis is used to derive the equations of fluctuating hydrodynamics on large length and time scales; the level of fluctuations is shown to be thermodynamically consistent with the equation of state of an isothermal, ideal gas.

## REFERENCES

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