8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

THE INVESTIGATION OF THE STABILITY OF SYSTEMS SUBJECTED TO STOCHASTIC PARAMETRIC FORCES IN THE FORM OF MARKOV PROCESSES

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Key Words: *Stability, Liapunov exponents, Stochastic Markov Processes, Statistical Moments.*

ABSTRACT

Load-bearing components of different structures are exposed to an action of loads which have a stochastic nature (e.g., wind and seismic loads, forces induced by waves, moving loads etc.). A reliable and economic design of such components requires an adequate mechanical modeling of the structural characteristics, which takes into account all uncertainties involved. One of the main problems to estimate the safety of similar structures is connected with the analysis of their stability. The behavior of linear elastic systems can be described by a system of stochastic differential equations with respect to nodal or generalized displacements. It can be made, for example, with help of variational methods (the finite element method, Bubnov-Galerkin method etc.). Stochastic parameters of these equations are assumed in the form of stationary processes with rational spectral densities. Because input and output processes of the formulated problem are Markov processes, then the probability density function (PDF) can be obtained from Fokker-Planck-Kolmogorov (FPK) equation. For the solution of FPK equation the Bubnov-Galerkin method is used. In the accordance with it PDF is sought in the following form

$$p(t, x_1, x_2, ..., x_n) = \sum_{i_1=1}^{n_1} \sum_{i_2=1}^{n_2} ... \sum_{i_n=1}^{n_n} f_{i_1 i_2 ... i_n}(t) \varphi_{i_1 i_2 ... i_n}(x_1, x_2, ..., x_n),$$

where t is the time, n is the number of space coordinates x_i in the FPK equation, $f_{i_1 i_2 ... i_n}(t)$ are generalized functions, $\varphi_{i_1 i_2 ... i_n}(x_1, x_2, ...x_n)$ are basic functions. Functions $f_{i_1 i_2 ... i_n}(t)$ and $\varphi_{i_1 i_2 ... i_n}(x_1, x_2, ...x_n)$ must satisfy initial and boundary conditions respectively. In such a case finally, we have a system of ordinary differential equations with respect to functions $f_{i_1 i_2 ... i_n}(t)$. With help of PDF, obtained in this way, statistical moments of desired quantities and then the values of Liapunov exponents are found. On the base of these exponents the stability of the considered system is estimated. The results are compared with exact analytical results for some simplest problems and with results, found with help of the numerical simulation of input random processes and the subsequent numerical solution of stochastic differential equations, describing the perturbed behavior of the system. This comparison shows that the mentioned results are near enough.
Acknowledgement - This work is supported by the Russian Foundation for Basic Researches (Grant 06-01-00095) which is gratefully acknowledged.