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APPLICATION-SPECIFIC ERROR CONTROL IN INVERSE IDENTIFICATION

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ABSTRACT

In this contribution we will employ the idea of adaptive modeling in the context of a two-step computation consisting of (i) a parameter identification problem where the identified parameters are used as inputs in (s) a subsequent simulation. Hence, in the result from the simulation in (s), we can expect two sources of error; the classical discretization error arising from the introduction of an FE-approximation in (s), and errors in the identified parameters stemming from (i).

In a slightly more formal setting, we want to analyze the system

$$\text{(i)}: \qquad p = \arg\left[\min_{q\in P} \mathcal{F}(q, u^{(\text{i})})\right], \quad \text{subject to} \quad a^{(\text{i})}(q, u^{(\text{i})}, v) = l^{(\text{i})}(v)\ \forall v \in V^{(\text{i})} \tag{1}$$

$$\text{(s)}: \qquad a^{(\text{s})}(p, u^{(\text{s})}, v) = l^{(\text{s})}(v)\ \forall v \in V^{(\text{s})}, \quad \text{giving} \quad \mathcal{Q}(u^{(\text{s})}). \tag{2}$$

In the parameter identification problem (i), the parameter $p \in P$ is determined from some sort of physical experiment. For example, p may represent a set of discrete material parameters, or a spatially varying function describing inhomogeneous material properties. The experimental setup is modeled by the state equation $a^{(\text{i})}(p, u^{(\text{i})}, v) = l^{(\text{i})}(v)$ where the response $u^{(\text{i})}$ depends on p. The objective function $\mathcal{F}$ is typically a least-squares functional measuring the discrepancy between the computed response and the corresponding (experimentally) observed response. The parameter p that minimizes $\mathcal{F}$ is used in the subsequent computation (s) to compute the goal quantity $\mathcal{Q}(u^{(\text{s})})$ (sometimes referred to as "quantity of interest") via the solution of the state equation $a^{(\text{s})}(p, u^{(\text{s})}, v) = l^{(\text{s})}(v)$.

We introduce suitable FE-approximations for the state variables in standard fashion, denoted $u_h^{(\text{i})} \in V_h^{(\text{i})} \subset V^{(\text{i})}$ and $u_h^{(\text{s})} \in V_h^{(\text{s})} \subset V^{(\text{s})}$, respectively. Moreover, we can also introduce a similar approximation of the parameter as $p_h \in P_h \subset P$ such that the discretized version of the system reads

$$\text{(i)}: \qquad p_h = \arg\left[\min_{q_h\in P_h} \mathcal{F}(q_h, u_h^{(\text{i})})\right], \quad \text{subject to} \quad a^{(\text{i})}(q_h, u_h^{(\text{i})}, v) = l^{(\text{i})}(v)\ \forall v \in V_h^{(\text{i})} \tag{3}$$

$$\text{(s)}: \qquad a^{(\text{s})}(p_h, u_h^{(\text{s})}, v) = l^{(\text{s})}(v)\ \forall v \in V_h^{(\text{s})}, \quad \text{giving} \quad \mathcal{Q}(u_h^{(\text{s})}). \tag{4}$$

The interpretation (and motivation) of $P_h \subset P$ depends on the specific application. For example, if p represents an arbitrary function describing a spatial inhomogeneity, we may introduce an approximation

p_h of FE-type in terms of nodal values and basis functions. Another example is that of a model hierarchy, where a simple material model such as the Neo-Hooke model (P_h) is obtained by a suitable model restriction of the more general Ogden model (P). We remark that even in the case without explicit approximation of the parameters, i.e. $P_h = P$, we still get an error $e_p = p - p_h$ due to the discretization $V_h^{(\mathrm{i})} \subset V^{(\mathrm{i})}$. From the viewpoint of the simulation (s) with the goal quantity $\mathcal{Q}$, we can perceive the error $e_p = p - p_h$ as a model error, while $e_u = u^{(\mathrm{s})} - u_h^{(\mathrm{s})}$ is the "classical" discretization error.

We employ the framework of goal-oriented a posteriori error estimation based on a dual problem to estimate the error $\mathcal{E} \stackrel{\mathrm{def}}{=} \mathcal{Q}(u^{(\mathrm{s})}) - \mathcal{Q}(u_h^{(\mathrm{s})})$, cf. Ainsworth and Oden [1], for the identification problem, Meidner and Vexler [2], Johansson et al. [3]. With the aid of the pertinent dual solutions and residuals, we can trace the error contributions to the different approximations defined by $V_h^{(\mathrm{i})}$, $V_h^{(\mathrm{s})}$ and P_h, respectively. Here, the two problems (i) and (s) are truly separated, and we may adopt the approach by Larson et al. [4] for estimating error transport between different problems. The relative magnitude of the different (estimated) contributions can be used as feedback for adaptive improvement of the respective approximations.

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