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ANALYSIS AND NUMERICAL APPROACH OF A PIEZOELECTRIC CONTACT PROBLEM

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ABSTRACT

Currently there is a considerable interest in contact problems involving piezoelectric materials, i.e. materials characterized by the coupling between the mechanical and electrical properties. This coupling, in a piezoelectric material, leads to the appearance of electric potential when mechanical stress is present and, conversely, mechanical stress is generated when electric potential is applied. The first effect is used in mechanical sensors, and the reverse effect is used in actuators, in engineering control equipments.

In this paper we study a quasistatic frictional contact problem for a piezoelectric body, when the foundation is conductive; our interest is to describe a physical process in which both contact, friction and piezoelectric effect are involved, to show that the resulting model leads to a well-posed mathematical problem, and to provide the numerical approach of the solution.

The physical setting is as follows. A body made of a piezoelectric material occupies the domain $\Omega \subset \mathbb{R}^d$ $(d = 2, 3)$ with a smooth boundary $\partial\Omega = \Gamma$. We assume a partition of Γ into three open disjoint parts Γ_1, Γ_2 and Γ_3, on the one hand, and a partition of $\Gamma_1 \cup \Gamma_2$ into two open parts Γ_a and Γ_b, on the other hand. The body is acted upon by body forces of density $\boldsymbol{f}_0$ and has volume free electric charges of density q_0. Moreover, the body is clamped on Γ_1 and surface tractions of density $\boldsymbol{f}_2$ act on Γ_2. We also assume that the electrical potential vanishes on Γ_a and a surface electrical charge of density q_b is prescribed on Γ_b. In the reference configuration the body may come in contact over Γ_3 with an obstacle, the so called foundation. The contact is frictional and is modeled with the normal compliance condition and a version of Coulomb's law of dry friction. Also, there may be electrical charges on the part of the body which is in contact with the foundation and which vanish when contact is lost. The process is assumed to be quasistatic and the mechanical properties of the material are assumed to be viscoelastic.

We derive the variational formulation of the problem which is given by a system coupling an evolutionary variational inequality for the displacement field with a nonlinear time-dependent variational equation for the electric potential field. Then we prove the existence of a unique weak solution to the model, under a smallness assumption on the surface conductance. The proof is based on arguments of evolutionary variational inequalities and fixed points of operators, similar to those used in [2,3].

Next, we consider a discrete scheme of the problem, obtained via the finite-element method, and prove its unique solvability. For the sake of generality we employ a numerical approach based on the augmented Lagrangian method [1], which permits to deal both with the unilateral and normal compliance contact condition. Then we use the discrete scheme as a basis of a numerical code for the problem, in which we develop a specific contact element. We need this element in order to take into account the coupling of the mechanical and electrical unknowns on the contact boundary condition.

In order to test the efficiency of our approach, we end our paper with a two-dimensional example. The setting is depicted in Figure 1, where $\Omega = [0,12] \times [0,4]$, $\Gamma_1 = \Gamma_b = (\{0\} \times [0,4]) \cup (\{12\} \times [0,4])$, $\Gamma_2 = \Gamma_a = [0,12] \times \{4\}$, and the potential contact surface is $\Gamma_3 = [0,12] \times \{0\}$. The body is subjected to the action of surface tractions $\boldsymbol{f}_2 = (0,-2)N/m^2$; the body forces and electric charges vanish, i.e. $\boldsymbol{f}_0 = \mathbf{0}\,N/m^3$, $q_0 = 0\,C/m^3$ and $q_b = 0\,C/m^2$; the gap between the body and the foundation is $g = 0.2m$ and the duration of the time interval of interest is $T = 0.5s$.

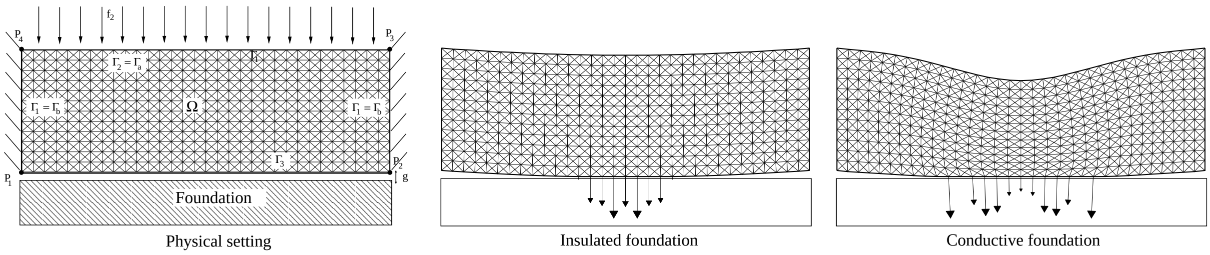

Figure 1: Physical setting and amplified deformed mesh with interface forces.

Our interest in this example is to study the influence of the electrical conductivity of the foundation on the contact process and, to this end, we consider the problem both in the case when the foundation is insulated and in the case when it is conductive. When the foundation is insulated there are no electric charges on Γ_3 and when the foundation is conductive, the normal component of the electric displacement field is assumed to be proportional to the difference between the potential of the foundation and body's surface potential. Our results are summarized in Figure 1 in which the deformed configurations and the frictional contact forces at the final time T are plotted. We can easily note that considering an electrically conductive foundation increases the deformations, the contact surface and the magnitude of friction forces. The algorithm was found to perform well, the convergence was rapid and the computations were good.

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