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ASYMPTOTICS OF RESONANT TUNNELING IN QUANTUM WAVEGUIDES OF VARIABLE CROSS-SECTION

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ABSTRACT

We consider electron motion in quantum waveguides with variable cross-sections. The narrows of the waveguide play the role of effective potential barriers for the longitudinal motion of electrons. Two narrows form a quantum resonator where a resonant tunneling may appear. It means that electrons with energy in a small range Δ pass through the resonator with probability near to 1. Various electronic devices (resonant transistors, key devices etc.) can be based on this phenomenon.

The Δ is rapidly decreasing as the diameter of narrows tends to 0, which presents difficulties for numerical modeling of the phenomenon. The full qualitative description of the phenomenon can be given only by asymptotic analysis.

We consider an infinite waveguide with two cylindric ends and two narrows of diameter ε. We give an asymptotic description of the electron wave propagation in such a waveguide as ε tends to zero. The wave number k is assumed to be sufficiently small, so only two normal modes exist in the waveguide at infinity: an incoming wave and an outgoing one.

The asymptotic formulas depend on the shape of narrows (in the limit as $\varepsilon \to 0$) of the waveguide. In the 2D case, we assume that a neighborhood of each narrow, in the limit as $\varepsilon \to 0$, coincides with a neighborhood of the vertex of two vertical angles of opening ω. While the openings of narrows are the same and the resonator is symmetric in a sense, the resonant energies (i.e., the energies k^2 at which the resonant tunneling occurs) are given by

$$k_r^2(\varepsilon) = k_0^2 + k_1^2 \varepsilon^{2\pi/\omega} + O(\varepsilon^{2\pi/\omega+2}),$$

where k_0^2 is an eigenvalue of the resonator (part of the waveguide between two narrows obtained after passing to the limit as $\varepsilon \to 0$), k_1^2 is a constant independent of ε, which can be found numerically. Near $k = k_r$, the transmission coefficient is of the form

$$T(k,\varepsilon) = \frac{1}{1 + Q\left(\frac{k^2 - k_r^2}{\varepsilon^{4\pi/\omega}}\right)^2},$$

Q being a positive constant independent of ε. From here we find the width of the resonant peak at half-height:

$$\Delta(\varepsilon) = \frac{2}{\sqrt{Q}}\varepsilon^{4\pi/\omega}.$$

To obtain these formulas we first construct an asymptotic representation of the corresponding wave function using the method of ”compound” asymptotics [1], [2]. The expression for the wave function is rather cumbersome; we do not present it here.

Analogous formulas (becoming more sophisticated) are valid for asymmetric waveguides where the resonator is not symmetric and (or) the narrows have distinct openings and diameters. In particular, in contrast to the symmetric case, the principal part of the asymptotics already shows that the maximum of the transmission coefficient is less than 1.

In the 3D case we assume that a neighborhood of each narrow, in the limit as $\varepsilon \to 0$, coincides with a neighborhood of the vertex of a cone. Asymptotic formulas for the basic characteristics of the process are analogous to those in the 2D case. The exponent $2\pi/\omega$ of ε must be replaced by the first eigenvalue of a problem on the base of the cone.

The method we used can be applied to multichannel scattering, in particular, to the scattering in waveguides with several ends.

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