8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

SPECTRAL DECOMPOSITION OF TRANSIENT VECTOR FLOW : DUFFING SYSTEM AND CHAOS

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Key Words: *Spectrum decomposition, Manifold, Foliation, Vector Flow.*

ABSTRACT

Chaotic dynamics have strongly been affected by the global structure of phase space. At the periodical steady state, the feature can be estimated by power spectra of converged periodic orbits. In non-autonomous systems, the power spectra of resonant frequencies and harmonics have an important role to distinguish the behavior. On the other hand, chaos has broadly spread power spectra except the discrete characterized spectra. The spectra include the information of global structure of phase space, even in the transient state directing to convergence. This report focuses on the decomposition of phase space of a nonlinear system based on the concept of spectrum partition [1]. The classical spectrum analysis possibly reveals the hidden feature of global structure related to power distribution and invariant sets [2]. Recently, the possibility of analysis on phase space topologies through periodic orbit [3].

The spectral partition is applied to Duffing system;

$$\begin{cases} \dot{x} &= y, \\ \dot{y} &= -ky - x^3 + A\cos t. \end{cases} \quad (1)$$

The parameters are set according to Ref. [4]. Here, the damping coefficient k is set at 0 and external force A at 0.2 for the demonstration. In the case, the system shows chaotic sea with resonant and harmonics solutions. Fig. 1 shows the distribution of components of power spectra on the initial value plane. For each point on the plane (x, y), the trajectories are integrated and the power specta are obtained. The contrast of the figure implies the strength of each component in 120 periods. The fundamental component has the highest contrast at the center of white region (see Fig. 1(a)). The 3rd harmonics also shows the highest contrast in the region where a resonant solution exists (see Fig. 1(b)). In the dark ring surrounding the resonant region, the 3rd component does not exist. In the chaotic sea, the component is low. The boundary of chaotic sea is not clearly distinguished but includes the information.

The 1/3 subsynchronous component exists at the boundary of fundamental resonant region and arounds the three fixed points of 1/3 (see Fig.1 (c)). Fig.1(d) shows the continuous componetnt except harmonics. The chaotic sea remarkably shows almost homogeneous distribution of dots. It implies the diffusion

of power spectrum shows globally even strength in the sea. However the boundary between the resonant region and the sea is not clearly obtained by the decomposition. This depends on the phase structure of the invariant sets [5]. The similar characteristics can also been observed in Lyapunov spectra distribution [6].

In this report, we applied a simple power spectrum analysis to understand the structure of phase space related to the chaotic sea in Duffing system. As for the system with a damping, it is also discussed that the invariant sets can be grasped by the spectrum analysis based on the detail decomposition at the boundary. We are going to analyze the stability of foliations by using the decomposition.

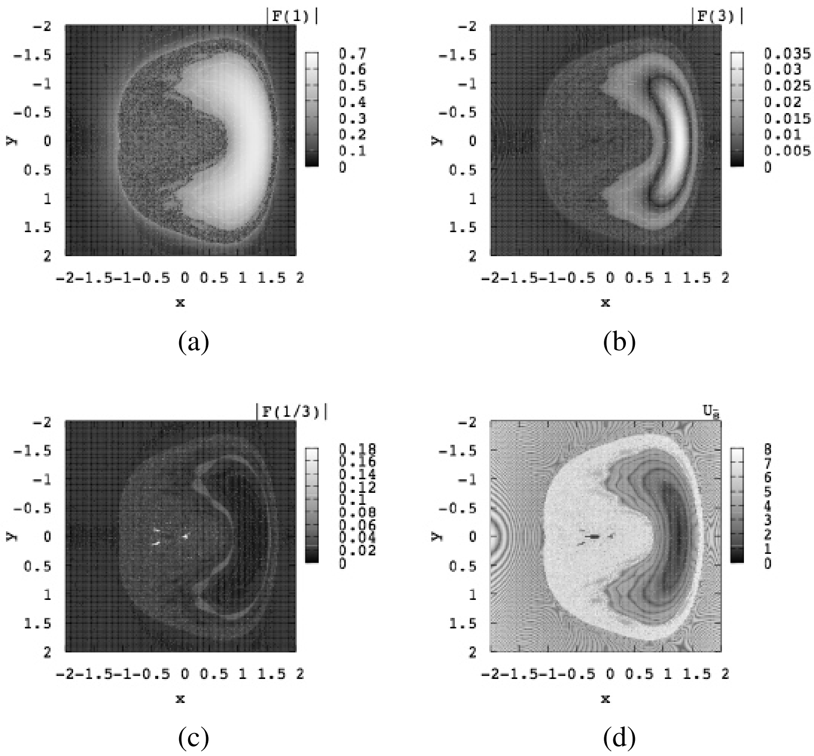

Figure 1: Distribution of power spectrum. (a) fundamental component, (b) 3rd harmonics, (c) 1/3 subsynchronous component, and (d) continuous power components harmonics.

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