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A UNIFYING FRAMEWORK FOR CONTACT PROBLEMS IN PLASTICITY

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ABSTRACT

In many applications involving several deformable bodies, frictional contact has to be taken into account as well as inelastic material behavior such as plasticity with isotropic/ kinematic hardening. Both kinds of problems lead to the mathematical structure of variational inequalities involving non-differentiable functions as well as constraints for the set of admissible solutions. In this talk, a unifying framework for the numerical treatment of these inequalities is presented.

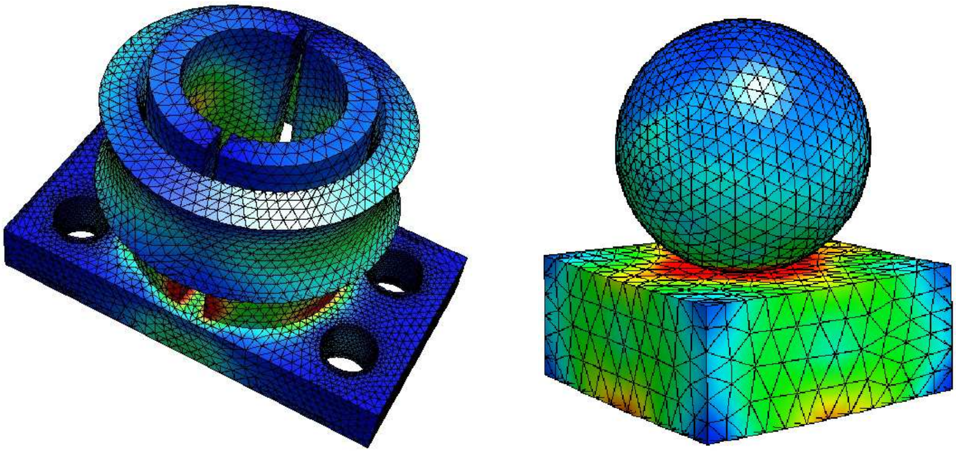

Figure 1: Effective stress results for 3D frictional contact problems

By introducing dual variables which correspond to the contact stresses or the deviatoric stresses, respectively, the problem can be transfered into a mixed formulation which allows for the application of the so-called primal-dual active set strategy. The inequality constraints arising from the non-penetration condition, the Coulomb friction law and the yield condition, are written as a set of non-smooth equations. Combined with the equilibrium equations, this leads to a system of nonlinear and nondifferentiable equations which can be solved in terms of a semi-smooth Newton method. If the dual variables are formulated in terms of dual basis functions, these additional degrees of freedom can be condensed out such that only a system of the original size has to be solved in each Newton step.

This unifying approach allows for a substantial reduction of computational time if the resulting nonlinear problems are solved inexactly, i.e. if the active sets are updated after each Newton iteration. This makes even complicated three-dimensional contact problems computationally feasible, as numerical examples with Coulomb friction and infinitesimal plasticity show (see Figure 1).

Furthermore, the extension of the above scheme to dynamic problems and large strains are presented. In the former case, the computations for the contact stresses are stabilized by employing a specially constructed mass lumping method; for the latter setting, the necessity to formulate the contact conditions with respect to the actual configuration and to implement it efficiently are discussed.

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