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NUMERICAL PREDICTION OF CRACK WIDTHS IN CONCRETE STRUCTURES

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ABSTRACT

The behavior and the durability of concrete structures is strongly influenced by cracking and by the width of existing cracks. In this contribution a numerical model for cracking of concrete is presented and applied to the numerical prediction of crack widths in anchor pull out tests and in tests on models of bridge cantilever slabs, strengthened by a concrete overlay.

The employed crack model is characterized by a modular structure. Basically, it consists of a combination of a smeared rotating crack model and a crack model, based on the strong discontinuity approach and formulated within the framework of embedded discontinuities [1]. Hence, it follows the idea proposed in [2]. A simple linear triangular element is used as the underlying finite element and the constitutive relationships are described for plane stress states within the framework of the theory of plasticity. Crack initiation is predicted by the Rankine criterion followed by the crack evolution described by a standard smeared rotating crack model. The crack direction can be predicted either by the maximum principal local strain or by the direction related to the maximum principal nonlocal strain. The latter choice allows to improve the numerical representation of the principal strain directions, i.e. to obtain a smoothing effect for the strains.

If the crack opening attains a prescribed threshold value, then a discontinuity in the displacement field

$$[\![\mathbf{u}]\!]_\Gamma = \zeta_n \, \mathbf{n} + \zeta_t \, \mathbf{t} \tag{1}$$

is embedded into the respective finite element with the crack opening ζ_n in the direction of the normal vector $\mathbf{n}$ of the discontinuity, predicted by the smeared rotating crack model, and ζ_t as the relative tangential displacement of the crack surfaces in the direction of the tangent vector $\mathbf{t}$ to the discontinuity with the initial value equal to zero. The further evolution of the embedded discontinuity accounts for both crack opening in the direction normal to the crack and relative tangential displacements of the crack faces with transfer of shear stresses across the crack faces. The respective interface laws are described by the yield conditions

$$f_n(\boldsymbol{\sigma}, q_n) = (\mathbf{n} \otimes \mathbf{n}) : \boldsymbol{\sigma} - q_n(\zeta_n, \zeta_t) \quad \text{and} \quad f_t(\boldsymbol{\sigma}, q_t) = |(\mathbf{t} \otimes \mathbf{n}) : \boldsymbol{\sigma}| - q_t(\zeta_n, \zeta_t) \tag{2}$$

together with the softening law for the tensile stress transferred across the discontinuity

$$q_n(\zeta_n) = f_t\, e^{-\zeta_n/\zeta_{n,u}} \quad \text{with} \quad \zeta_{n,u} = G_f/f_t\,, \tag{3}$$

where f_t and G_f are the tensile strength and the specific fracture energy, respectively, and the hardening law for the shear stress transferred across the discontinuity

$$q_t(\zeta_n, \zeta_t) = -\frac{f_c}{30} + [1.8\zeta_n^{-0.8} + (0.234\zeta_n^{-0.707} - 0.20)f_c]\,|\zeta_t| \tag{4}$$

with f_c representing the uniaxial compressive strength. The constitutive relation (4) was proposed in [3]. It holds for tangential slips $\zeta_t^0 \leq \zeta_t \leq \zeta_t^1$. For $\zeta_t \leq \zeta_t^0$ with ζ_t^0 determined by setting $q_t = 0$ in (4), the transferred shear stress across the crack faces is negligible, whereas for $\zeta_t \geq \zeta_t^1$ due to aggregate interlock compressive stresses will be transferred across the crack faces. In this case, according to Walraven and Reinhardt, $(3)_1$ is replaced by

$$q_n(\zeta_n, \zeta_t) = f_t\, e^{-\zeta_n/\zeta_{n,u}} + \frac{f_c}{20} - [1.35\zeta_n^{-0.63} + (0.191\zeta_n^{-0.552} - 0.15)f_c]\,|\zeta_t| \tag{5}$$

and ζ_t^1 is computed by setting $q_n = 0$ in (5). As soon as a discontinuity is embedded into a finite element, the crack direction is kept fixed. Then, there are two options available for the crack path. Either the discontinuities are embedded locally in the individual elements, which means that the crack path is not continuous across the common edges of neighboring elements, or continuity of the crack path is enforced by means of a crack tracking algorithm.

The model is applied to the numerical simulation of an anchor pull-out test and to the numerical simulation of model tests on damaged bridge cantilever slabs, which are strengthened by concrete overlays.

Several tests on the load carrying behavior of models of bridge cantilever slabs strengthened by concrete overlays were performed at the University of Innsbruck [4]. The numerical simulation of the tests consists of the numerical simulation of (i) the damage caused by loading of the original cantilever slab, (ii) the effects of shrinkage of the applied concrete overlay on the strengthened slab and (iii) cracking due to loading of the strengthened slab. The aim of the numerical analyses was to predict the crack widths and the stresses at the interface between the original cantilever slab and the concrete overlay.

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