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Efficient evaluation of stress intensity factors using a coupled BEM-SBFEM algorithm

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ABSTRACT

The scaled boundary finite element method (SBFEM) is gaining more recognition as an alternative to other element-based and meshless methods. Originally conceived as a tool for computing the dynamic stiffness of an unbounded domain [1], the method has since demonstrated greater versatility. One such area is its application to linear elastic stress analysis, particularly in fracture mechanics. The method works numerically in the circumferential direction making the usual piecewise polynomial approximation. However this is then coupled to an analytical solution in the radial direction. The method is especially useful in calculating stress intensity factors (SIFs) as they are functions of a displacement found semi-analytically rather than fully numerically [2].

The SBFEM is not without its drawbacks however. Its use in engineering domains containing holes and other similar features requires cumbersome substructuring [3]. The method also requires computationally expensive matrix manipulation, including matrix inversion and the solution of an eigenvalue system whose dimensions are twice the number of degrees of freedom of the model.

The boundary element method (BEM) has also been developed to solve fracture mechanics problems. However, it makes only piecewise polynomial approximations to functions that are not polynomial in nature. The dual BEM allows efficient boundary-only crack modelling [4], but requires the evaluation of hypersingular integrals and still uses piecewise polynomial shape functions.

A coupled BEM-SBFEM system provides for the efficient calculation of SIFs for complex domains. A domain is subdivided into two regions. The crack tip is modelled by a small SBFEM subdomain with few degrees of freedom, thus keeping SBFEM computations to a minimum. This subdomain is coupled to a generally larger BEM subdomain that models the rest of the domain, thus exploiting the BEM's ability to model geometric features efficiently.

Initial studies have shown that this coupled algorithm provides accurate SIFs with few degrees of freedom and can be applied directly to general engineering domains [5]. The convergence of the coupled method has also been demonstrated [6].

Since the SBFEM region contributes only a small number of degrees of freedom, no attempt is made in the current work to write the BEM matrices in a symmetric form in an equivalent stiffness matrix. Instead, the SBFEM governing equation is written in terms of traction, allowing direct coupling with the BEM system. This leads to issues related to the application of Dirichlet boundary conditions to nodes common to both subdomains. Like the conventional application of zero-displacement boundary conditions in the FEM, the SBFEM removes such degrees of freedom from its stiffness matrix. These degrees of freedom cannot be removed in this manner from the BEM matrices, and the result is an underdetermined coupled system.

This paper describes how this problem is addressed by the inclusion of additional equations formed by extra boundary element collocation in the BEM subdomain. Numerical behaviour of the method is demonstrated.

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