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NUMERICAL SIMULATION OF A THERMOMECHANICAL MODEL ARISING IN STEEL HARDENING

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ABSTRACT

We consider the following thermomechanical model arising in the analysis of steel hardening including phase transitions ([1,3-5])

$$-\nabla\cdot\sigma = f \text{ in } \Omega\times(0,T_{\mathrm{f}}) \tag{1}$$

$$\sigma = K\left(\varepsilon(u) - \theta q(z)I - \int_0^t \gamma(\theta,z,z_t)S\,\mathrm{d}\tau\right) \tag{2}$$

$$z_t = F(\theta,z,\sigma) \text{ in } \Omega\times(0,T_{\mathrm{f}}) \tag{3}$$

$$z(0) = z_0 \text{ in } \Omega \tag{4}$$

$$\alpha(\theta,z,\sigma)\theta_t - \nabla\cdot(k(\theta)\nabla\theta) + 3\kappa q(z)\theta\nabla\cdot u_t = H + b(\theta)|\nabla\varphi|^2 \tag{5}$$

$$+ (\rho L + \mathrm{tr}\,\sigma\theta\bar{q} + 9\kappa\theta^2 q(z)\bar{q})z_t + \gamma(\theta,z,z_t)|S|^2 \text{ in } \Omega\times(0,T_{\mathrm{f}}) \tag{6}$$

$$\theta(0) = \theta_0 \text{ in } \Omega \tag{7}$$

$$\nabla\cdot(b(\theta)\nabla\varphi) = 0 \text{ in } \Omega\times(0,T_{\mathrm{f}}) \tag{8}$$

where $\Omega\subset\mathbb{R}^N$, $N=2$ or 3 (the workpiece) is a bounded, connected and Lipschitz-continuous open set; T_{f} is the final time of observation; σ is the stress tensor; $\varepsilon(u)=\frac{1}{2}(\nabla u+\nabla u^T)$ is the strain rate tensor; f is a given external force; u is the displacement field; θ is the temperature; $z=(z_1,z_2)$, z_1 and z_2 are the phase fractions ([1,2,6]) of austenite and martensite, respectively, whereas $q(z)=q_1z_1+q_2z_2+q_0(1-z_1-z_2)$, and q_i represents the expansion coefficient of austenite ($i=1$), martensite ($i=2$) and of the original mixture of phase fractions ($i=0$); $F=(F_1,F_2)$ gives the phase fractions model; $k(\theta)$ is the thermal conductivity; $b(\theta)$ is the electrical conductivity; ρ is the density; $\kappa=\frac{1}{3}(3\lambda+2\mu)$ is the bulk modulus of elasticity, λ and μ being the Lamé coefficients; $L=(L_1,L_2)$ is the latent heat;

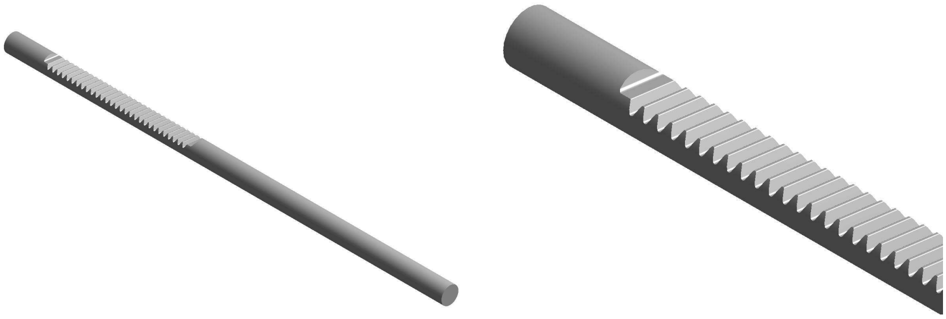

Figure 1: Car steering rack. Left, the workpiece. Right, tooth component.

$\bar{q} = \nabla_z q$; $S = \sigma - \frac{1}{3}\operatorname{tr}\sigma I$, that is, the trace free part of the stress tensor σ; the term $\int_0^t \gamma(\theta, z, z_t) S \,\mathrm{d}\tau$ models the transformation induced plasticity; H is a source term. Finally, α is of the form

$$\alpha(\theta, \sigma, z) = \rho c_\varepsilon - \operatorname{tr}\sigma q(z) - 9\kappa q(z)^2\theta,$$

where the constant c_ε is the specific heat capacity at constant strain. In this setting, the unknowns are the displacement field u, the phase fractions z, the temperature θ and the electrical potential φ. Equations (1-8) are supplied with suitable boundary conditions.

This model appears in steel hardening by means of heating-cooling treatments. The goal is to produce martensite in certain critical parts, and just there, in the workpiece. Usually, these parts correspond to particular structural components whose surface is going to be highly stressed during its mechanical lifetime. This is the case of the car steering rack (see Figure 1).

We have performed some numerical simulation of this model using finite elements in space and finite difference in time. These simulations can be applied in order to control the whole system so that one can have martensite where needed and, at the same time, minimize the distortions of the workpiece related to the heating-cooling treatment.

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