5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

High order non-periodic homogenization for seismic wave propagation in layered media and for the 2D SH case.

*** Yann Capdeville[1], Laurent Guillot[1] and Jean-Jacques Marigo[2]**

[1] Institut de Physique du Globle de Paris, CNRS
Case 89, 4 place Jussieu
75252 Paris, cedex 05, France
capdeville@ipgp.jussieu.fr

[2] Laboratoire de Modélisation en Mécanique, Université Paris 6
4 place Jussieu
75252 Paris, cedex 05, France

Key Words: *Seismology, Computing Methods, Homogenization, direct and inverse problem*

ABSTRACT

In many cases, in the seismic wave propagation modeling context, scales much smaller than the minimum wavelength are present in the earth model we wish to propagate in. For many numerical methods, like the spectral element method, these small scales need to be meshed in order to be accurate, which is a challenge leading to high numerical cost. For the inverse problem present in seismic tomography, understanding the interaction of small scales (that can not be imaged) with large scales (that can be imaged) is an important issue. The purpose of the work presented here is to understand and to build the effective medium and equations allowing to average the small scales of the original medium without losing the accuracy of the wavefield computation. Being able to do so will also be very important for seismic tomography by allowing to build a consistent parametrization for the inverse problem. For this kind of problems, two scale homogenization in periodic and random media is a well-known effective method [1], [2]. Nevertheless, application of two scale homogenization to non-periodic deterministic media is still a challenge.

We first apply high order two scale homogenization to periodic layered earth models. It appears that the order 0 homogenization gives the result that was obtained by Backus in 1962 [3] which implies that order 0 homogenized model is transversely isotropic even though the original model is isotropic. It also appears the order 0 is not enough to obtain surface waves with correct group and phase velocities and that higher order homogenization terms up to 2 are often required. High order homogenization implies to modify the wave equation and the boundary conditions. We show how to extend the theory from the periodic case to the non-periodic case. Examples in periodic and non-periodic medium are given. The accuracy of the results obtained by homogenization are checked against normal mode solution computed in the original media and shows a good agreement [4] (see figure 1). Applications to numerical method like the spectral element method [5] will be presented and the numerical difficulties linked to the new boundary conditions discussed.

The extension from layer media to media with properties varying rapidly in all direction is well known for periodic media case. Nevertheless, the extension to non-periodic deterministic media is difficult and the and the generalization of the solution found in the non-periodic layer media case is not straightforward. We will present preliminary results in that direction for the 2S SH case.

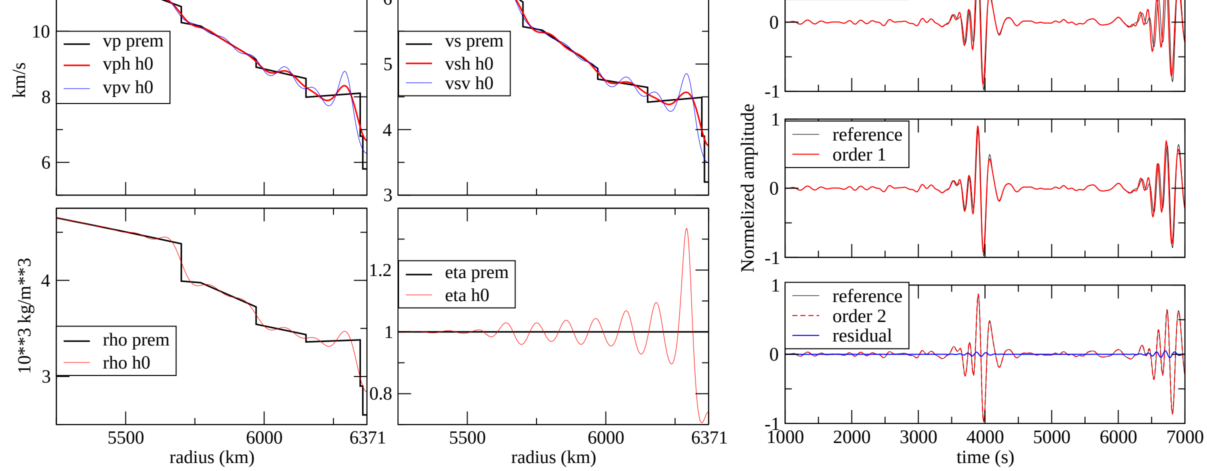

Figure 1: Homogenization example in the spherically symmetric earth model PREM. On the left if shown the Isotropic PREM model and its homogenized version the order 0 (h0). V_{ph} and V_{pv} (vertical and horizontal P wave speed), V_{sh} and V_{sv} (vertical and horizontal S wave speed), density and η for both models are represented as a function of the radius. For the Isotropic PREM, the horizontal and vertical wave speed are the same. It appears clearly that, even if the original model is isotropic, the order 0 homogenized is anisotropic. On the right is presented the vertical component of the displacement due to a vertical force at 221 km depth recorded at an epicentral distance of $132°$. The "reference" solution is computed in the PREM model and homogenized solutions at the order 0, 1 and 2 are shown. It is clear that the order 0 homogenization is not accurate enough for the surface waves, but a good accuracy can be achieved with the order 2. For the inverse problem, this result is interesting because it shows that it is useless to try to obtain a sharp model with this kind of data.
.

REFERENCES

[1] E. Sanchez-Palencia. "Non homogeneous media and vibration theory". N. 126, Lecture Notes in Physics, Springer, 1980.

[2] G. C. Papanicolaou and S.R.S. Varadhan. "Boundary value problems with rapidly oscillating random coefficients". Proceedings of Conference on Random Fields, Esztergom, Hungary, 1979.

[3] G.E. Backus. "Long-Wave Elastic Anisotropy Produced by Horizontal Layering". *J. Geophys. Res.* Vol. **67** 4427–4440, 1962.

[4] Y. Capdeville and J. J. Marigo. "Second order homogenization of the elastic wave equation for non-periodic layered media *Geophys. J. Int.*, Vol. 170, pp 823–838, 2007.

[5] Y. Capdeville and J. J. Marigo. "Shallow layer correction for Spectral Element like methods" *Geophys. J. Int.*, accepted, 2007.