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SPECTRAL NUMERICAL METHODS FOR MAXWELL'S EQUATIONS ON NON-STRUCTURED MESH

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ABSTRACT

For a long time, Maxwell's equations were mainly solved in the time-harmonic domain. The evolution of radar techniques showed the limit of this formulation which can only treat monochromatic sources. Both engineers and researchers were then motivated to use equations in the time domain which can take into account large frequency sources in one resolution. The first and most popular approximation of Maxwell's equations in the time domain was provides by the Yee's scheme, commonly called FDTD (Finite Difference in the Time Domain) by enginees, which is basically a centered second order finite difference approximation of Maxwell's equation.

Although easy to implement, FDTD has some difficulties to treat complex geometries. In fact, the staircase approximation of curved boundary can produce spurious reflections which can substantially pollute the solution. On the other hand, finite element methods (FETD) have the major drawback of producing a n-diagonal (n can grow up to several tens in 3D) mass matrix which must be inverted at each time-step, which is a serious handicap for FETD versus FDTD whose mass matrix is the identity matrix. This mass matrix does not present any difficulty to time harmonic problems, for which even the stiffness matrix must be inverted. For this reason, industry was reluctant to use FETD for a long time and FDTD remains the reference for Maxwell's equations in the time domain for 40 years!

The mass lumping technique is an efficient alternative to mass matrix inversion. However, this technique was well known for lower order continuous (or H^1) elements but not obvious for higher-order approximations. A first step towards a general mass lumping technique was made by Hennart et al. [1] which proposed to use Gauss-Lobatto quadrature formulas to get mass lumping for continuous quadrilateral or hexahedral finite elements. Besides mass lumping, these formulas ensure to keep the order induced by the finite element approximation. First introduced for ODE or parabolic problems, this technique was extended to the wave equation by Cohen et al. [2] and later renamed spectral element methods [3].

The problem of the mass matrix inversion was solved for the wave equation and remained a challenging problem for Maxwell's equations.A natural approach is provided by extending spectral element techniques to edge (or $H(curl)$) elements. This was done by Cohen et al. for orthogonal meshes for the first family [4] and for any mesh for the second family of edge elements of any order [5].

Due to the storage of the stiffness matrix, even by using mass lumping techniques, FETD remained much more expensive than FDTD in terms of storage and, to a lesser extent, in computational time. This ultimate problem was solved by using a mixed $H(curl) - L^2$ formulation of Maxwell's equations based on $H(curl)$-conform definition of the $curl$ operator in both spaces [5]. This technique provides a local definition of the stiffness matrices which induces a substantial gain of storage. A detailed presentation of all these techniques can be found in [6].

Unfortunately, although H^1 spectral elements and H^1 and $H(curl)$ triangular and tetrahedral elements behave quite well for any mesh, $H(curl)$ spectral elements present important parasitic waves for very distorted meshes, which are often used in industrial problems. For this reason, discontinuous Galerkin methods appeared as an efficient alternative for Maxwell's equations. First introduced by Hesthaven [7] for tetrahedra, this approach was adapted by Cohen et al. [8] to the spectral element point of view, which provided a low-storage as well as fast method to solve Maxwell's equations. This approach seemed to deal better with parasitic waves but eigenvalues considerations showed that such waves were however present in this method. All these remarks motivated us to discuss the numerical dissipation terms which can attenuate parasitic waves.

An extended presentation of this work can be found in [9].

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