

## USING VOLUME METRICS AS AN ALTERNATIVE TO PENALTY OR FINITE ELEMENT METHODS FOR MODELLING CONTACTS

\* Yves Gonthier<sup>1</sup>, Christian Lange<sup>1</sup> and John McPhee<sup>2</sup>

<sup>1</sup>Canadian Space Agency  
6767 route de l'Aéroport  
Saint-Hubert, Québec, Canada J3Y 8Y9  
E-mail: {Yves.Gonthier|Christian.Lange}@space.gc.ca

<sup>2</sup>University of Waterloo  
Waterloo, Ontario, Canada N2L 3G1  
E-mail: mcphee@uwaterloo.ca

**Key Words:** *Contact Dynamics, Impact, Multibody Systems, Volumetric Properties, Robotics, Winkler Foundation.*

### ABSTRACT

The Canadian Space Agency (CSA) is investigating new methods for modelling the interaction forces between robotic systems and their environments. In particular, the CSA is concerned with producing accurate and reliable simulations of tele-operated manipulators to insert payloads into specially designed worksites, and ensuring that the operations are performed smoothly and without causing damage. Traditionally, the force interactions are predicted using one of the three following methods.

On one hand, the colliding bodies can be assumed to be infinitely stiff. The associated rigid body model must be used in conjunction with an impact hypotheses, such as Newton's, Poisson's, and Stronge's [1]. Hereby, the equations of motion are derived by balancing the system's momenta before and after the impact, i.e., without explicitly considering contact compliances. This allows for great simplifications since changes in velocity become instantaneous, and can be calculated without integrating accelerations over the contact period. However, [5] shows that the contact stiffness and the duration of the impact phase greatly influences the stability of robotic systems under discrete-time control. Hence, rigid body models provide an inadequate reference for validating the performance of robotic systems in contact.

In contrast, penalty methods describe the rate-dependent normal and tangential compliance relations over time, and are usually based on Hertz theory. These second methods rely on elastic half-space theory to find contact stiffness properties of the bodies. Here, the stiffness is directly related to the interfering geometries and to the material properties of the bodies. A nonlinear spring model is then used to calculate the magnitude of the contact forces throughout the contact phase. However, the solution of the Boussinesq integral can only be found in the cases where the geometries are simple, and the area of the contact patch is assumed to be small in comparison to the size of the bodies.

The third option is to rely on continuum models, which model the mechanical interaction of the bodies in detail as a function of the material properties. These models are often implemented using Finite Element Methods (FEM) [6], and their application is restricted to non real-time simulation because of the large computational overhead associated with FEM. The body geometries are transformed into a mesh, and the FEM computes the deformation of each element of the mesh. Obviously, the accuracy of the prediction is directly related to the meshing pattern used and to the resulting number of elements.

This work presents another method that could be considered “in-between” the latter two options: the *volumetric approach*. The volumetric contact model is derived from first principles assuming a simplified mechanical behaviour of the materials: the Winkler elastic foundation model [4, 3]. In [2], the contact force expression is derived by defining an expression for the contact pressure as a function of the body geometries. The latter is integrated over the contact area, and the overall body-to-body force is obtained. It was shown that this contact force can be expressed in terms of the volumetric properties of the *volume of interference* between the two bodies, defined as the volume spanned by the intersection of the two undeformed geometries of the colliding bodies. The properties of interest are: the volume of the volume of interference, the position of its centroid, and the inertia tensor about the centroid. The volumetric approach is in fact a continuum model by nature, i.e., the effect of each infinitesimal element of the contact surface is taken into account in the overall contact force prediction. However, in its implementation, the volumetric contact model becomes a penalty model: the model corresponds to a nonlinear spring whose force is proportional to the volume of the volume of interference.

The volume metrics are obtained either analytically or numerically. In the former case, because the mathematical framework is simple, closed-form solutions can be found for a larger range of geometries than with the elastic half-space theory. For the latter case, the volume metrics are found by discretizing the volume of interference into small cubes called *voxels*, and extracting the volume metrics from these simple primitives. It is worthwhile to note that the discretization is not performed *a-priori* as with FEM, but is done on-line, with a dynamically selected resolution. Furthermore, the processed geometries are not restricted to have polygonal descriptions, and can be described using an exact mathematical representation [7]. Finally, the algorithms used to extract the volume metrics are highly suitable for parallel processing by nature. Hence, the computer implementation of the simulation can benefit from the huge computational power available in the latest high performance computing platforms, which are based on parallel multicore architectures. The simulation infrastructure as well as some contact dynamics simulation results for objects with complicated geometries will be presented.

## References

- [1] C. Glocker. *Set-Valued Force Laws — Dynamics of Non-smooth Systems*. Springer, Berlin, 2001.
- [2] Y. Gonthier. *Contact Dynamics Modelling for Robotic Task Simulation*. PhD thesis, University of Waterloo, Waterloo, Ontario, Canada, 2007.
- [3] G. Hippmann. An algorithm for compliant contact between complexly shaped bodies. *Multibody System Dynamics*, 12(4):345–362, December 2004.
- [4] K. L. Johnson. *Contact Mechanics*. Cambridge University Press, London, 1985.
- [5] J. Kövecses, L. L. Kovács, and G. Stéphan. Dynamics modelling and stability of robotic systems with discrete-time force control. *Archive of Applied Mechanics (Ingenieur Archiv)*, 77(7):293–299, May 2007.
- [6] M. A. Puso and T. A. Laursen. A mortar segment-to-segment frictional contact method for large deformations. *Computer Methods in Applied Mechanics and Engineering*, 193:4891–4913, 2004.
- [7] J.-F. Rotgé and J. Farret. Universal solid 3D format for high performance urban simulation. In *Urban Remote Sensing Joint Event*, pages 1–10, Paris, France, April 11–13 2007. Proceedings of the IEEE.