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STABLE MANIFOLD COMPUTATIONS

IN A NON-SMOOTH DYNAMICAL SYSTEM

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ABSTRACT

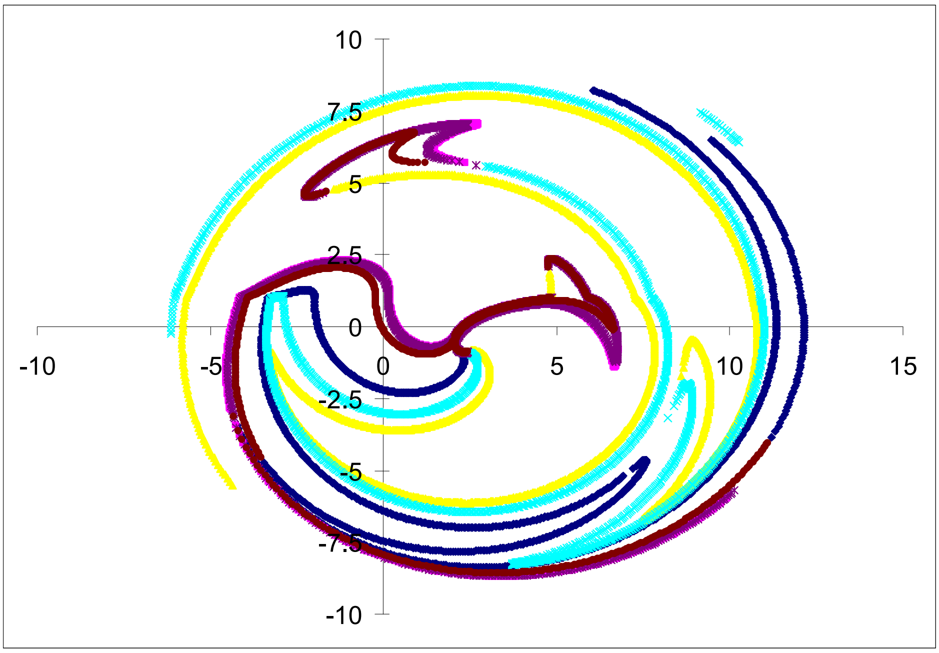
Stable manifolds of saddle points are important in defining the dynamics of smooth nonlinear dynamical systems [1]. The stable manifold theorem for a fixed point states that there are local stable and unstable manifolds tangent to the eigenspaces of the linearised system at the fixed point. The global stable (and unstable) manifold is given by the union of backward (and forward) mappings in time of the local manifold. If we restrict our attention to two-dimensional Poincaré maps of three-dimensional flows, at a saddle point of the map, which corresponds to a saddle-limit cycle of the flow, the linearised system will have one-dimensional stable and unstable eigenspaces. At the saddle the local manifolds are tangent to the eigenvectors and in a neighbourhood of the saddle the local manifolds are therefore approximated by these eigenvectors [2]. A numerical procedure to compute the global stable manifolds derives directly from what we just said: a fixed point of a Poincaré map is located with standard numerical algorithms [2] and its eigenvectors are computed. If it is a saddle point then an eigenvector is larger than one in modulus and the other is smaller. The stable eigenspace corresponds to the eigenvalue smaller than one. A number of points approximately lying on the stable manifold are then generated by choosing them close to the saddle in the direction of the stable eigenvector. The global stable manifold can be traced by integrating backward in time such a set of points. Numerical issues can affect the computation of the stable manifold; in particular the 'stretching' of the manifolds may reduce the degree of accuracy with which the manifold is reconstructed, but most of these problems have been successfully overcome for smooth systems [2].

It is apparent that several difficulties will arise if the vector field is non-smooth [3, 4]: it may be difficult to define the associated linearised system, in particular if the saddle-cycle has a sliding branch, as it is common in friction oscillators; then the solution may be not unique in backward time and therefore questions may arise on how to numerically integrate backward in time; moreover the dimension of the phase space may vary during the motion of the system. The present paper describes a way to compute the stable manifold of a saddle-like limit cycle of a non-smooth dynamic

system of Filippov's type [3]. The method will be applied to the simple friction oscillator presented in [5].

It is important to observe that our method does not make use of concepts such as linearised system, Jacobian matrix and eigenvalues, the definition of which would be problematic in a non-smooth system. Our approach, based only on the accurate knowledge of the unstable orbit therefore, is different from that presented in [6], which is applied to non-invertible non-smooth two-dimensional maps.

The paper will discuss different cases to which the proposed method can be applied with different levels of success. The reasons for that will be discussed.



Stable manifold of a saddle-limit cycle of the simple friction oscillator presented in [5]. Different colours correspond to branches of the manifold computed from different 'initial conditions'.

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