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Discontinuous Galerkin Methods for the Chemotaxis Model and Closely Related Biomedical Problems.

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ABSTRACT

In this work, first, we propose a family of interior penalty discontinuous Galerkin methods for the Keller-Segel chemotaxis model. This model is described by a system of two nonlinear PDEs: a convection-diffusion equation for the cell density coupled with a reaction-diffusion equation for the chemoattractant concentration. It has been recently shown that the convective part of this system is of a mixed hyperbolic-elliptic type, which may cause severe instabilities when the studied system is solved by straightforward numerical methods. Therefore, the first step in the derivation of the proposed methods is made by introducing the new variable for the gradient of the chemoattractant concentration and by reformulating the original Keller-Segel model in the form of a convection-diffusion-reaction system with a hyperbolic convective part. We then design interior penalty discontinuous Galerkin methods for the rewritten Keller-Segel system. Our methods employ the central-upwind numerical fluxes, originally developed in the context of finite-volume methods for hyperbolic systems of conservation laws.

We prove error estimates for the proposed high-order discontinuous Galerkin methods. Our proof is valid for pre-blow-up times since we assume boundedness of the exact solution. We also show that the blow-up time of the exact solution is bounded from above by the blow-up time of our numerical solution.

In the numerical tests that will be presented, we first compare three different discontinuous Galerkin schemes applied to the Keller-Segel model:

1) primal discontinuous Galerkin methods applied to the original formulation of the Keller-Segel model,
2) primal discontinuous Galerkin methods with the standard upwind numerical fluxes for the reformulated Keller-Segel model and,
3) the new discontinuous Galerkin methods.

We show, that compare to the new discontinuous Galerkin methods, the first two schemes fail to give the accurate, oscillation free solutions for the classical Keller-Segel chemotaxis model.

Second, we consider application of the proposed discontinuous Galerkin schemes, to some biomedical problems.

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