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An Anisotropic Sparse Grid Stochastic Collocation Method for Partial Differential Equations with High-Dimensional Random Input Data

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ABSTRACT

This talk proposes and analyzes an anisotropic sparse grid stochastic collocation method for the approximation of statistical quantities related to the solution of partial differential equations with random coefficients and forcing terms (input data of the model). This method is an extension of the Sparse Grid Stochastic Collocation method analyzed in [7], which consists of a Galerkin approximation in the space variables and a collocation, in probability space, on sparse tensor product grids utilizing either Clenshaw-Curtis or Gaussian knots. Even in the presence of nonlinearities, the collocation approach leads to the solution of uncoupled deterministic problems, just as in the Monte Carlo method.

Our previous sparse collocation procedure is very effective for problems whose input data depend on a moderate number of random variables, which "weigh equally" in the solution. For such an isotropic situation the displayed convergence is faster than standard collocation techniques built upon full tensor product spaces.

On the other hand, the convergence rate deteriorates when we attempt to solve highly anisotropic problems, such as those appearing when the input random variables come e.g. from Karhunen-Loève-type truncations of "smooth" random fields. In such cases, a full anisotropic tensor product approximation may still be more effective for a small or modest number of random variables. However, if the number of random variables is large, the construction of the full tensor product spaces becomes infeasible, since the dimension of the approximating space grows exponentially fast with respect to the number of random variables in the problem.

Instead, this work proposes the use of anisotropic sparse tensor product spaces constructed from the Smolyak algorithm utilizing suitable abscissas. This approach is particularly attractive in the case of truncated expansions of random fields, since the anisotropy can be tuned to the decay properties of the

expansion. We present optimal *a priori* and *a posteriori* procedures for tuning the anisotropy of the sparse grids to each given problem. These procedures have been shown to be very effective in quantifying uncertainty for several complex stochastic systems of equations. In this talk we will highlight examples from computational mechanics; in particular, fluid mechanics and engineering sciences.

This work also provides a rigorous convergence analysis of the fully discrete problem and demonstrates: (sub)-exponential convergence in the asymptotic regime and algebraic convergence in the pre-asymptotic regime, with respect to the total number of collocation points. Numerical examples illustrate the theoretical results and are used to compare this approach with several others, including the standard Monte Carlo. In particular, for moderately large dimensional problems, the sparse grid approach with a properly chosen anisotropy seems to be very efficient and superior to all examined methods.

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