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HYPER REDUCTION OF FINITE STRAIN ELASTO-PLASTIC MODELS

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ABSTRACT

We propose a general method of solving reduced-order equations related to models involving internal variables. The proposed approach is based on the classical snapshot Proper Orthogonal Decomposition (POD) [1] and on an original Petrov Galerkin formulation of the reduced governing equations. A reduced-order model (ROM) is a simplified model that derived from a detailed model by assuming that the state variables belong to a subspace of the state space related to the detailed model. Hence the number of independent state variables (the order of the model) is reduced. The purpose of a model reduction method is to build a basis of this subspace. The POD model reduction method has been used in a wide range of disciplines including signal analysis and simulation in fluid mechanics or in oceanography [2]. This method comes from the Karhunen-Loève expansion [3, 4] developped for statistical analyses. A POD basis is a basis of the state subspace spanned by state simulations eventually related to different model modifications. The availability of ROMs can greatly facilitate the solution of optimization problems [5].

The aim of this work is to propose a new method to compute the reduced state variables related to a known POD basis. The classical Galerkin formulation of the reduced governing equation related to a POD basis is not efficient for medium size elasto-plastic problems (about 20000 Degrees of freedom). In such problems the computational cost related to the local integration of the nonlinear constitutive equations is very important. It can constitute 80% of the total computational cost. The classical POD basis applied to primal variables (the displacements) does not affect the efficiency of the integration of the constitutive equations. To overcome this drawback we propose a Petrov-Galerkin formulation related to a Reduced Integration Domain (RID). This RID is created by selecting few elements of the mesh to perform the local integration of the constitutive equations. This is a generic approach coined Hyper Reduction, which have been successfully applied to simplify nonlinear thermal transient models [6].

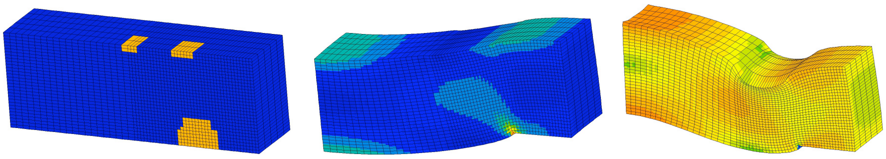

Figure 1: The reduced integration domain (red elements on the left), the first and the second shape functions of the POD bases related to the displacements and internal variables.

The novelty of this work is the extension of the Hyper Reduction method to the problems involving internal variables. Such variables are widely used in solid mechanics to depict viscoelasticity, viscoplasticity, plasticity and also continuum damage mechanics. Due to the restriction of the integration to the RID, the Hyper Reduction method does not provide an estimation of the internal variables over the entire mechanical system. An additional POD basis is proposed to extrapolate the computed internal variables. Therefore the proposed approach enables to forecast all the state variables of the entire mechanical system.

A 3D finite strain elasto-plastic model is considered to illustrate the capabilities of the Hyper reduction method. The problem of concern is a simple forging process of a rectangular workpiece. The detailed model involves 15000 hexaedrons with 8 nodes per element, 1560000 scalar internal variables and 45198 degrees of freedom related to nodal displacements. The Hyper Reduced Order Model involves 5 shape functions to depict the displacements and 914 selected elements to perform the prediction of the reduced state variables.

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