

## MULTIPLE POINT CONSTRAINTS IN FINITE ELEMENT ANALYSIS OF TWO-DIMENSIONAL ELASTIC CONTACTS

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### ABSTRACT

In this paper a finite element procedure is developed to analyze normal and partial slip contacts between two elastic cylinders. Contact conditions are transformed into multiple point constraints (called MPCs henceforce) for nodal displacements. These constraint conditions together with the finite element system equations are solved using the technique of Lagrange multipliers [1]. Stresses and displacements so obtained satisfy stress and displacement contact conditions. The method developed in this article may be used to obtain the contact length, lengths of the stick and the slip regions, and the normal and the tangential contact stresses [2].

Two methods to convert stress contact conditions into MPCs are suggested. In the first method stress contact conditions are replaced by conditions on nodal forces, which are then transformed into MPCs for nodal displacements. In the second method, stress contact conditions are written in terms of nodal stresses whose values are obtained by extrapolating stresses at Gaussian points, and then these conditions are transformed into MPCs for nodal displacements. It is found in this study that both methods work well for frictionless contacts. For frictional contact problems only the second method is recommended [2].

Cases treated in this article include normal contacts with and without friction, and tangential contacts with partial slip. Numerical results include size of contact region, lengths of the slip and the stick zones, and normal and tangential stresses in the contact region. Results for partial slip contacts show strong interactions between the normal problem and the tangential problem, even for cylinders of identical materials.

## REFERENCES

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