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ENERGY-MOMENTUM ALGORITHMS FOR NONLINEAR COUPLED THERMO-ELASTODYNAMICS

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ABSTRACT

Classical time-stepping algorithms, like Newmark schemes and their variations, are well-known by now to exhibit a number of serious limitations when applied to nonlinear finite deformation problems in solid dynamics. For example, algorithms that are known to be unconditionally stable in the linear range show numerical instabilities when applied to nonlinear problems. Often these instabilities are characterized by an uncontrolled growth of the energy of the system. Notably, the same difficulties appear in inelastic problems, where a non-negative energy dissipation is to be expected based on physical considerations. Simply put, these physical properties are not inherited by the discrete numerical system. This situation has motivated the formulation of new numerical algorithms that exactly capture these conservation/dissipation properties of the energy. Similarly, capturing exactly the conservation laws of linear and angular momenta appears as a basic requirement for the correct simulation of the systems of interest in nonlinear solid and structural dynamics.

We present in this contribution the development of these ideas for the case of nonlinear coupled thermoelastic solids. The resulting systems exhibit the aforementioned dissipative structure as a direct consequence of the second law of thermodynamics in the considered coupled thermomechanical framework. The energy dissipation does not only apply to the standard internal energy but to the so-called canonical free energy. In fact, it is this quantity that defines the contractive (i.e. dissipative) character of the mathematical evolution problem in terms of the mechanical and thermal variables (e.g. displacements/velocities and temperature). We argue that, in this context, it is this precise contractive structure in the canonical free energy that needs to be preserved by the numerical integration algorithms, in addition to the simpler-to-accomplish balance of energy and momentum conservation laws. We present then a new energy-momentum algorithm that does exhibit these properties, hence inheriting a strong notion of nonlinear stability. The algorithm relies crucially on the definition of the appropriate approximation formula for the entropy in the solid, in addition to existing conservation formulas for the stresses. We present complete analyses of these properties, as well as several numerical simulations that illustrate them and the improved numerical performance thus obtained.