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REMARKS ON THE SOLUTION OF THE INITIAL VALUE PROBLEM FOR ANISOTROPIC FINITE ELASTOPLASTICITY CONSIDERING VARIOUS FORMULATIONS OF THE MATERIAL MODEL

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ABSTRACT

Practical material models as well as their efficient and stable numerical integration provide the basis for the reliable computation of the stress response of components and structures to an external mechanical loading. Commonly, nonlinear field problems of solid mechanics are numerically processed using spatial discretization methods, like the finite element method (FEM), solving the boundary value problem and time discretization procedures treating the embedded initial value problem (IVP).

Following generally accepted axioms and assumptions the authors developed a phenomenological, thermodynamically consistent material model for large anisotropic elastoplastic deformations based on a substructure concept (cf. [1]). Within this context, the constitutive relations are finally defined by a system of differential and algebraic equations (DAE). The numerical integration of the DAE represents the solution of the IVP.

As usual in elastoplasticity, the material model originally comprises of a stress relation in rate formulation,

$$\dot{\boldsymbol{T}} = \frac{1}{2} \underset{4}{\boldsymbol{D}} \cdot\cdot \dot{\boldsymbol{C}} - \underset{4}{\boldsymbol{D}} \cdot\cdot \lambda \frac{\partial F}{\partial \boldsymbol{T}} - \lambda \left(\boldsymbol{T} \frac{\partial F}{\partial \boldsymbol{T}} \boldsymbol{C}^{-1} + \boldsymbol{C}^{-1} \frac{\partial F}{\partial \boldsymbol{T}} \boldsymbol{T} \right) \tag{1}$$

evolutional equations for the internal variables modeling the hardening behavior, and the yield condition F. Here $\boldsymbol{T}$ denotes the 2nd Piola-Kirchhoff stress tensor, $\boldsymbol{C}$ the right Cauchy-Green tensor, and $\underset{4}{\boldsymbol{D}}$ the fourth order hyperelastic material tensor.

Due to the necessary time discretization this approach is associated with an incremental stress computation. It will be shown that, within this context, the accuracy of stress values essentially deteriorates with increasing load steps. Consequently, we substitute the usual stress formulation by the flow rule (cf. [1]):

$$\dot{\boldsymbol{C}}^p \boldsymbol{C}^{p-1} \boldsymbol{C} + \boldsymbol{C} \boldsymbol{C}^{p-1} \dot{\boldsymbol{C}}^p = 4\lambda \frac{\partial F}{\partial \boldsymbol{T}} \,. \tag{2}$$

Therefore, we include the plastic strain tensor $\boldsymbol{C}^p$ instead of the stress tensor into the set of unknown variables of the IVP. Stresses are explicitly computed from a hyperelastic material law depending on the elastic strain tensor. This strategy is advantageous for more reliable stress results at large load steps. Additionally, a more stable convergence behavior has been observed.

As an alternative to the plastic strain tensor the authors studied the accuracy and the convergence behavior of the solution of the IVP considering a DAE with an evolutional equation for the plastic part of the deformation gradient $\boldsymbol{F}^p$

$$\dot{\boldsymbol{F}}^p = \lambda \boldsymbol{F}^p \boldsymbol{C}^{-1} \frac{\partial F}{\partial \boldsymbol{T}} \tag{3}$$

instead of the stress formulation. Within this context, the so-called Mandel stress tensor (see e. g. [2]) is defined. This procedure simplifies the mathematical structure of the system to be solved as well as the computation of substructure-based variables which are suitable for the analysis of texture development. Additionally, the consideration of an anisotropic hyperelastic part of the material model will be provided.

In former investigations, the elastoplastic material model was usually integrated eliminating the plastic multiplier based on the consistency condition. Especially stimulated by publications of Simo and other authors (cf. [3]), various return mapping methods have been developed and successfully applied. Starting from an elastic predictor return algorithms provide an iterative solution of the plastic multiplier satisfying the yield condition. Following, the actual stresses and the internal variables can be calculated easily based on the time discretization of the evolutional equations.

For the numerical treatment of the IVP within a finite element approach we prefer a simultaneous solution of the complex DAE (cf. [4,5]) which distinguishes itself by a higher efficiency and accuracy compared with the classical methods mentioned above. An additional benefit of this strategy results from the efficient numerical determination of the consistent material matrix. The time discretization of the DAE is realized based on the generalized implicit single-step scheme

$$y_{n+1} = y_n + (\alpha \, f_{n+1} + (1-\alpha) \, f_n) \, \Delta t \tag{4}$$

for an ordinary first order differential equations. The time increment $\Delta t = t_{n+1} - t_n$ represents the current load step, and the parameter $\alpha \in [0, 1]$ controls the accuracy and convergence rate of the solution of the IVP. Applying the scheme (4) to the DAE representing the material law a system of nonlinear algebraic equations can be derived. This system is solved by means of Newton method.

The presented numerical strategy for the solution of the elastoplastic IVP has been implemented into the author's in-house FE-code. Some examples illustrating and comparing its accuracy, stability as well as efficiency with respect to the various formulations of the DAE are discussed.

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