8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

ASYMPTOTIC ANNIHILATION ALGORITHM FOR NON-LINEAR DYNAMICS

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Key Words: *Integration in time, non-linear dynamic, asymptotic annihilation algorithm, numerical damping.*

ABSTRACT

Algorithms that eliminate any non-zero response in a high-frequency mode after one time step are referred as asymptotic annihilation algorithms [1]. Numerical dissipation is desirable since the high-frequency response is typically an artefact of structural discretisation (mesh refinement). In the present paper, a very efficient asymptotic annihilation algorithm [2] in terms of accuracy and computational efforts is employed to integrate non-linear equilibrium equation.

The step-by-step integration algorithm to be considered in this paper takes into account the following fourth-order Hermitian operators [1]

$$6y_i + 2\Delta t y_i^I - 6y_{i-1} + 4\Delta t y_{i+1}^I - \Delta t^2 y_{i+1}^{II} = 0 - \frac{1}{12}\Delta t^4 y_i^{IV}$$

$$2\Delta t y_i^I - 2y_{i+1}^I + 2\Delta t^2 y_{i+1}^{II} - \Delta t^3 y_{i+1}^{III} = 0 - \frac{1}{3}\Delta t^4 y_i^{IV}$$

where Δt is the time step, y is the function to be integrated (the time derivative notation by roman numeral as exponent is employed).

Starting point of the non-linear dynamic analysis can be expressed in matrix notation by

$$My^{II} + Cy^I + Ky = f + n$$

with mass matrix M, damping matrix C, stiffness matrix K, given vector forces f and any non-linear parts represented by vector n. Taking into account the above equilibrium equation an its derivative those Hermitian operators can be rewrite as

$$\begin{bmatrix} A & B \\ D & E \end{bmatrix} \begin{Bmatrix} \Delta t y_{i+1}^I \\ y_{i+1} \end{Bmatrix} = \begin{bmatrix} F & G \\ H & J \end{bmatrix} \begin{Bmatrix} \Delta t y_i^I \\ y_i \end{Bmatrix} - \begin{bmatrix} P & L \\ S & R \end{bmatrix} \begin{Bmatrix} \Delta t \left(f_{i+1}^I + n_{i+1}^I \right) \\ f_i + n_i \end{Bmatrix}$$

where

$$A = \Delta t C + 4M \quad B = \Delta t^2 K - 6M \quad D = -2M - \left(2M + \Delta t C\right)\Delta t M^{-1} C + \Delta t^2 K \quad E = -\left(2M + \Delta t C\right)\Delta t^2 M^{-1} K$$

$$F = H = -2M \qquad G = -6M \qquad J = P = 0 \qquad L = S = \Delta t^2 I \qquad R = -2\Delta t^2 I - \Delta t^3 C M^{-1}$$

which is the recurrence equation for the proposed algorithm. It is important to point out that for lumped model (matrices M and C diagonal) the factorization of this recurrence equation involves the factorization of matrix $E - DA^{-1}B$ only (this is the factorization of a matrix of order n).

In order to illustrate the numerical efficiency, a classical non-linear single degree of freedom [3] governed by the equation

$$my^{II} + cy^{I} + \frac{2ES}{\ell} y = f - (2N_0 - 2ES)\frac{y}{\sqrt{\ell^2 - y^2}}$$

where N_0 is the magnitude of the pre-stressed force, ES is the elastic product and ℓ is the length of the bar, is considered.

The following table compares the non-linear relative period error obtained against the Newmark method results. It is important to register that these algorithms are fourth-order local truncation error as explicitly one can see from the Hermitian operators above presented and second-order global truncation error (period elongation).

Relative Period Error

Δt	$T_n/10$	$T_n/25$	$T_n/50$	$T_n/100$
Annihilation	$1.18\ 10^{-2}$	$3.14\ 10^{-4}$	$9.72\ 10^{-3}$	$1.01\ 10^{-4}$
Newmark	$2.22\ 10^{-2}$	$3.10\ 10^{-3}$	$7.43\ 10^{-4}$	$2.03\ 10^{-4}$

The results shown indicate that although the proposed algorithm presents the desired annihilation property the accuracy is in competition with the most popular Newmark scheme.

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