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3D PROBLEM FOR AN INTERFACE CRACK UNDER HARMONIC LOADING

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ABSTRACT

The elastodynamic response of intra- and inter-component cracks to an elastic wave is a topic of long standing interest in the study of wave propagation. A large number of papers are devoted to the wave diffraction problem for a crack in homogeneous material, see [1, 2]. At that, the overwhelming majority of authors neglect the contact interaction of crack faces in spite of its evident significance. Only the more recent studies are focused on accounting for non-linear effects due to the crack closure under dynamic loading, see, for example, the review by Guz and Zozulya [3]. However, elastodynamic investigation of inter-component cracks received considerably less attention than the case of cracks in homogeneous materials.

In the current study we investigate an *interface* crack between two dissimilar homogeneous isotropic elastic half-spaces undergoing harmonic loading. The conditions of continuity for displacements and stresses are satisfied at the bonding interface region. In the absence of body forces, the stress-strain state of both domains is defined by the Lamé dynamic equations of the linear elasticity, and the Sommerfeld radiation-type condition is imposed at infinity on the displacements.

It should be emphasised that the opposite faces of the existing crack interact with each other under deformation, affecting significantly the stress and strain fields. The nature of the contact interaction between two crack surfaces is very complex. The shape of the contact region is unknown beforehand; it changes in time under deformation of the body and must be determined as a part of solution. The complexity of the problem is further compounded by the fact that the contact behaviour is very sensitive to the material properties of two contacting surfaces and the type of the external loading. Such dependences make the contact crack problem *highly non-linear*. The resulting process is a steady-state periodic, but not a harmonic one. As a result, components of the stress-strain state cannot be represented as a function of coordinates multiplied by an exponential function. Therefore we expand the components of the displacement and traction vectors into exponential Fourier series [3].

In order to take the contact interaction of crack faces into account we assume there is no interpenetration of the opposite crack faces and the contact force is unilateral. Thus the

Signorini constraints are imposed for normal components of displacements and forces. We also assume that the contact interaction satisfies the Coulomb friction law, which means that the opposite crack faces remain immovable with respect to each other in the tangential direction until they are held by the friction force, however, as soon as the magnitude of the tangential contact forces reaches a certain limit depending on the friction coefficient, the crack faces begin to move: the slipping motion occurs.

The detailed procedure for deriving the system of boundary integral equations (BIE) was given in [4–6]. It was shown that the system of BIE for displacements and tractions at the interface can be obtained using the Somigliano dynamic identity. The integral kernels are the fundamental solutions of the dynamic problem in the frequency domain, evaluated by the consecutive differentiation of the Green's displacement tensor [1–3].

The system of BIE derived in [4–6] was solved numerically by the collocation method with the piecewise constant approximation of the given and unknown functions. The hyper-singular integrals, present in the resulting system of linear algebraic equations, were treated in the sense of the Hadamard finite part.

The distributions of the displacements and tractions at the bimaterial interface and the crack surface were obtained and analysed for the case of a penny-shaped crack under the longitudinal wave. The stress intensity factors (opening and shear modes) were computed for different values of the wave frequency and different properties of the material. Note that the obtained results are in a very good agreement with the analytical static solution for an interface crack [7] and with the numerical dynamic solution for a crack in the homogeneous body [3]. When the frequency tends to zero, the solution of the dynamic problem tends to the analytical solution of the corresponding static problem for all material combinations. Furthermore, when the difference between properties of half-spaces tends to zero, the dynamic solution tends to the one obtained for the homogeneous material.

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