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RETROSPECTIVE OF INTRINSIC METHODS IN THE THEORY OF THIN AND ASYMPTOTIC SHELLS

M. C. Delfour

Centre de recherches mathématiques, Université de Montréal,
CP. 6128, succ. Centre-ville, Montréal (Qc), Canada H3C 3J7. delfour@crm.umontreal.ca

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ABSTRACT

Many hypersurfaces ω in $\mathbf{R}^N$ can be viewed as the boundary or a subset of the boundary Γ of an open subset Ω of $\mathbf{R}^N$. In such cases the *oriented distance function* b_Ω to the underlying set Ω completely describes the surface ω: its (outward) normal is the gradient ∇b_Ω, its first, second, third, ..., and N-th fundamental forms are $\nabla b_\Omega \otimes \nabla b_\Omega$, its Hessian $D^2 b_\Omega$, $(D^2 b_\Omega)^2$, ... and $(D^2 b_\Omega)^{N-1}$ restricted to the boundary Γ ([8], [11, Chapter 8, § 5]). In addition, a fairly complete intrinsic theory of Sobolev spaces on $C^{1,1}$-surfaces is available in [5].

In the theory of thin shells, the asymptotic model, when it exists, only depends on the choice of the *constitutive law*, the *midsurface*, and the subspace of the space of solutions that properly handles the loading applied to the shell. A central issue is how rough this midsurface can be to still make sense of asymptotic *membrane shell* and *bending equations* without ad hoc mechanical or mathematical assumptions. This is possible for a general $C^{1,1}$-midsurface with or without boundary such as a sphere, a donut, or a closed reservoir. Moreover, it can be done without local maps, local bases, and Christoffel symbols via the purely intrinsic methods developed by Delfour and Zolésio starting in 1992 with [9] and in a number of subsequent papers. The key paper [2] uses intrinsic methods in the asymptotic analysis of three models of thin shells for an arbitrary linear 3D constitutive law. They all converge to asymptotic shell models that consist of a coupled system of two variational equations. They only differ in their resulting effective constitutive laws. The first equation yields the generally accepted classical *membrane shell equation* and the Love-Kirchhoff terms. The second is a generalized *bending equation*. It explains that convergence results for the 3D models were only established for plates and in the bending dominated case for shells. From the analysis of the three models, the richer $P(2,1)$-model turns out to be the most pertinent since it converges to the right asymptotic model with the right effective constitutive law. We also show in [3] that models of the Naghdi's type can be obtained directly from the $P(2,1)$-model by a simple elimination of variables without introducing the a priori assumption on the stress tensor $\sigma_{33} = 0$. Bridges are thrown with classical models using local bases or representations.

Those results are completed in [3] with the characterization of the space of solution for the $P(2,1)$ thin shell model and the space of solutions of the asymptotic membrane shell equation in [4]. This characterization was only known in the case of the plate and uniformly elliptic shells. In [6], a new choice of the projection leads to the disappearance of the coupling term in the second asymptotic equation. After reduction of the number of variables, this changes the form of the second equation to achieve the complete

decoupling of the membrane and bending equations without the classical plate or bending dominated assumptions. In the second part of [6], we present a dynamical thin shell model for small vibrations and investigate the corresponding dynamical asymptotic model. They complete [2] and connect with most existing results in the literature thus confirming the pertinence and the interest of the methods we have developed. Extensions of the $P(2,1)$-model have also been developed for piezoelectric shells [7, 1] and a complete decoupling of the membrane and bending equations is also obtained.

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