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STRESS-INTEGRATION ALGORITHMS FOR GEOMECHANICS PROBLEMS INVOLVING LARGE DEFORMATIONS

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In nonlinear finite element analysis, loads are usually applied in increments and the corresponding incremental displacements are obtained by solving the global equilibrium equations. The incremental strains can be computed from the incremental displacements in the usual way. A set of ordinary differential equations must then be solved to find the stress increment based upon the known strain increment. This system of equations may be written as:

$$\dot{\sigma}_{ij} = C^{ep}_{ijkl} \cdot \dot{\varepsilon}_{kl} \quad , \quad \dot{\kappa}_i = B_i\left(\sigma, \kappa\right) \cdot \dot{\lambda} \tag{1}$$

where σ is the true (Cauchy) stress tensor, C^{ep} is the constitutive matrix, ε denotes the strain tensor, κ represents a set of hardening parameters, B is a function derived from the hardening laws, $\dot{\lambda}$ is a positive scalar called the plastic multiplier. For large deformation analysis, the stress-strain relations can no longer be expressed as simply as equation (1), since the components of the true stresses may change due to possible rigid body motion. In other words, the principle of objectivity requires that rigid body motion must induce no extra strain in the material. Objectivity is usually satisfied by introducing a frame-independent stress-rate into the stress-strain relations. The most commonly used stress rates in large deformations problems of geomechanics are the Jaumann stress rate and the Truesdell stress rate (see e.g. [1]). Introducing, for instance, the Jaumann stress rate into the constitutive equations, one can write:

$$\sigma_{ij}^{t+\Delta t} = \sigma_{ij}^{t} + \int_0^{\Delta\varepsilon_{ij}} d\sigma_{ij} = \sigma_{ij}^{t} + \int_0^{\Delta\omega_{kl}} \left(\sigma_{ik} \cdot d\omega_{jk} + \sigma_{jk} \cdot d\omega_{ik}\right) + \int_0^{\Delta\varepsilon_{kl}} C_{ijkl}(\sigma, \kappa) \cdot d\varepsilon_{kl} \tag{2}$$

where ω is the spin tensor. Equation (2) shows that the effect of rigid body motion must be introduced to the constitutive equations during stress integration. This effect can be introduced either before, after or during integration of the constitutive equations, providing three alternative algorithms. However, no theoretical advantage exists for selecting any one of these three strategies. Moreover, it appears that the advantages and disadvantages of one strategy over the others have not yet been reported in the literature (see [1]). This study attempts to compare alternative algorithms for integrating stress-strain relations in a large deformation analysis.

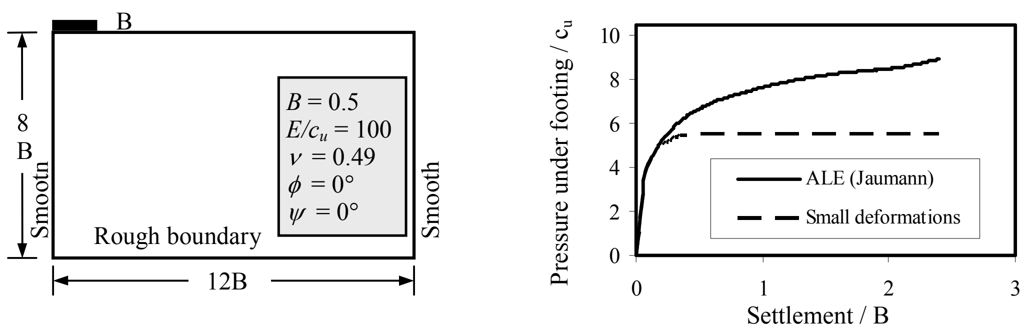
Algorithm 1 includes the following steps: 1-Enter with stresses σ_{ij}^{t}, the strain increment $\Delta\varepsilon_{ij}$ and the spin tensor increment $\Delta\omega_{ij}$. 2- Correct σ_{ij}^{t} for rigid body rotation (2nd term on RHS of (2)) to find $\tilde{\sigma}_{ij}^{t}$. 3- Integrate stress-strain relations (3rd term on RHS of (2)) with $\tilde{\sigma}_{ij}^{t}$ and $\Delta\varepsilon_{ij}$ to find $\sigma_{ij}^{t+\Delta t}$. Note that in step 3 a stress-integration scheme used in small deformations analysis may be invoked. Algorithm 2 performs the following steps: 1-Enter with stresses σ_{ij}^{t}, the strain increment $\Delta\varepsilon_{ij}$ and the spin tensor increment $\Delta\omega_{ij}$. 2- Integrate stress-strain relations (3rd term on RHS of (2)) with σ_{ij}^{t} and $\Delta\varepsilon_{ij}$ to find $\tilde{\sigma}_{ij}^{t}$. 3-Correct $\tilde{\sigma}_{ij}^{t}$ for rigid body rotation (2nd term on RHS of (2)) to find $\sigma_{ij}^{t+\Delta t}$. Algorithm 3 integrates the stress-strain relations and the correction due to rigid body motion simultaneously over the increment. This algorithm is based on the original work developed by Sloan [2]. Due to lack of space, this algorithm is not explained in detail here. Further details can be found in references [1] and [3].

To investigate the performance of three alternative stress-integration algorithms described above, a rigid rough footing on an undrained soil layer represented by an associated Tresca model is considered. Refer to Figure 1 for geometry, material properties and the predicted load-displacement curve. This load-displacement curve was obtained using the Arbitrary-Lagrangian-Eulerian (ALE) method based on incorporation of the Jaumann stress rate into the analysis (see [1] for more details).

The load-displacement curves obtained using all three algorithms are more or less similar to the plot shown in Figure 1.b. Thus in terms of accuracy, all algorithms provide essentially the same solution. To study the efficiency of each algorithm, the CPU times and the number of iterations necessary to achieve equilibrium in each analysis are shown in Table 1. This table shows that Algorithm 1 requires less equilibrium iterations compared to Algorithms 2 and 3. Although the difference between the performance of Algorithms 1 and 2 is not significant, both clearly outperform Algorithm 3. No significant advantage between Algorithm 1 and Algorithm 2 is demonstrated by this example. However, these two algorithms outperform Algorithm 3 in terms of efficiency.

Algorithm	CPU time (Sec)	Total iterations
1	5687	1999
2	6138	2176
3	16039	2040

Table 1. CPU time and total equilibrium iterations of alternative stress integration algorithms.



a. Geometry and material

b. Load-displacement response

Figure 1. Analysis of rigid rough footing on an undrained layer of soil.

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