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NUMERICAL METHODS FOR 3D COULOMB'S FRICTION BASED ON NONSMOOTH NEWTON'S METHOD AND NONLINEAR COMPLEMENTARITY FORMULATIONS

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ABSTRACT

3D Coulomb's friction The article is devoted to the study of numerical methods for 3D Coulomb's friction based on Nonsmooth Newton's method and Nonlinear Complementarity Problem (NCP) formulations. Let us introduce the finite–dimensional (eventually after a FEM space–discretization) dynamics of a mechanical systems subjected to 3D Coulomb's friction with unilateral constraints. Taking into account possible nonsmooth evolutions, the system is formulated in terms of a measure differential inclusion

$$\begin{cases} M(q(t))dv + N(q(t), v^+(t))dt + F_{\text{int}}(t, q(t), v^+(t))\, dt = F_{\text{ext}}(t)\, dt + dr \\ v^+(t) = \dot{q}^+(t) \\ U^+(t) = \nabla^T g(q) v^+(t), \quad U^-(t) = \nabla^T g(q) v^-(t) \\ dr = \nabla g(q) dR \\ \text{If } g(q) \leqslant 0 \text{ then } C^* \ni [U_{\text{N}}(t^+) + \mu \|U_{\text{T}}(t^+)\| + eU_{\text{N}}(t^-), U_{\text{T}}] \perp dR \in C \end{cases} \tag{1}$$

where q, v are the coordinates (or the displacements) and the velocities, dv, dr are differential measures, dt is the Lebesgue measure M, N are respectively the mass matrix, the gyroscopic acceleration, $F_{\text{int}}, F_{\text{ext}}$ the internal forces and the external forces, g defines the constraints or the local coordinates at contact, U, dR the local relative velocity and the measure of contact efforts. The subscript $\cdot_{\text{N}}$ and $\cdot_{\text{T}}$ defines their normal and tangential components at contact, C is the second order Coulomb's cone, *i.e.* $C = \{R, \|R_{\text{T}}\| \leqslant \mu |R_{\text{N}}|\}$ and C^* its dual where μ is the coefficient of friction, and e is the newton's coefficient of restitution. More details can be found in [1].

Moreau's time-stepping scheme. The system is linearized by Newton's method and is time–discretized by Moreau's scheme [2] (see [1] for details). One gets

$$\begin{cases} U_{k+1} = \widehat{W} P_{k+1} + U_{\text{free}} \\ \left.\begin{array}{l} \widehat{U}^\alpha_{k+1} = \left[U^\alpha_{\text{N},k+1} + e^\alpha U^\alpha_{\text{N},k} + \mu^\alpha\, ||U^\alpha_{\text{T},k+1}||, U^\alpha_{\text{T},k+1}\right]^T \\ \mathbf{C}^{\alpha,*} \ni \widehat{U}^\alpha_{k+1} \perp P^\alpha_{k+1} \in \mathbf{C}^\alpha \end{array}\right\} \forall \alpha \in I_a(\tilde{q}_{k+1}) \end{cases} \tag{2}$$

where $W = \nabla^T g(\tilde{q}_{k+1})\widehat{M}^{-1}\nabla g(\tilde{q}_{k+1})$ is the Delassus operator and I_α the index set of the forecast constraints, U_{free} the velocity without constraints and $\tilde{q}_{k+1}$ a prediction of the position. At each time step the time–discretized linear one–step nonsmooth problem (2) which is a second order cone complementarity problem (SOCCP) has to be solved for the unknowns U_{k+1} the discrete relative velocity and P_{k+1} the discrete impulse.

Nonsmooth Newton's methods The pioneering work of Alart and Curnier [3] extends the standard Newton's method to the case of a nonsmooth but continuous functions. The formulation and the associated numerical method is based on an equation based formulation of the system (2) thanks to the projection operator proj onto the friction disk $D(\mu R_{\text{N}}) = \{R_{\text{T}}, \|R_{\text{T}}\| \leqslant \mu |R_{\text{N}}|\}$:

$$\begin{cases} U_{k+1} = \widehat{W} P_{k+1} + U_{\text{free}} \\ \left.\begin{array}{l} P^\alpha_{\text{N},k+1} = \text{proj}_{\mathbb{R}_+}(P^\alpha_{\text{N},k+1} - \rho^\alpha_{\text{N}}\,(U^\alpha_{\text{N},k+1} + e^\alpha U^\alpha_{\text{N},k})) \\ P^\alpha_{\text{T},k+1} = \text{proj}_{\widehat{\mathbf{D}}^\alpha(P^\alpha_{\text{N},k+1}, U^\alpha_{\text{N},k+1})}(P^\alpha_{\text{T},k+1} - \rho^\alpha_{\text{T}} \circ U^\alpha_{\text{T},k+1}) \end{array}\right\} \forall \alpha \in I_a(\tilde{q}_{k+1}) \end{cases} \tag{3}$$

where $\rho^\alpha_{\text{N}} > 0$, $\rho^\alpha_{\text{T}} \in \mathbb{R}^2_+ \setminus \{0\}$ for all $\alpha \in I_a(\tilde{q}_{k+1})$ and the modified friction disk is $\widehat{\mathbf{D}}^\alpha(P^\alpha_{\text{N},k+1}, U^\alpha_{\text{N},k+1}) = \mathbf{D}(\mu(\text{proj}_{\mathbb{R}_+}(P^\alpha_{\text{N},k+1} - \rho^\alpha_{\text{N}}\,(U^\alpha_{\text{N},k+1} + e^\alpha U^\alpha_{\text{N},k}))))$ for all $\alpha \in I_a(\tilde{q}_{k+1})$. We recall that $\cdot \circ \cdot$ is the Hadamard product of vectors.

In this work, this method is used as a reference method to compare with other formulations and with new algorithms. Moreover, we propose a line–search procedure to extend its convergence properties.

Nonsmooth Newton's methods based on Fischer–Burmeister function In [4], a NCP reformulation of the SOCCP (2) is derived in the form

$$0 \leqslant F(z) \perp z \geqslant 0 \tag{4}$$

with $F(z) = \widetilde{W}z + g(z) + q$ where $z, \widehat{W}, q$ and the smooth function g are defined in [4]. Once we obtain this formulation, we reformulate it in terms of nonsmooth equations thanks to the well–known Fischer–Burmeister function [5].

Results The comparison between these two approaches plus other projection/splitting formulations is studied on several standard examples. The efficiency of line–searches is analyzed. Finally, the algorithm is compared to the robust and widespread solver for NCP, the PATH solver [6]. Detailed of the implementation into SICONOS [7] `http://siconos.gforge.inria.fr` will be given.

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