8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering
(ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

High-Order Absorbing and Open Boundaries: Extensions and Improvements

*** Dan Givoli[1], Assaf Mar-Or[2] and Thomas Hagstrom[3]**

[1] Dept. of Aerospace Eng.
Technion
Haifa 32000
Israel
Email: givolid@aerodyne
.technion.ac.il

[2] Prog. of Applied Math.
Technion
Haifa 32000
Israel
E-mail:
pierrot@technion.ac.il

[3] Dept. of Math. & Stat.
The U. of New Mexico
Albuquerque, NM 87131
U.S.A.
E-mail:
hagstrom@math.unm.edu

Key Words: *Absorbing Boundary, Open Boundary, High Order, Artificial Boundary, Evanescent Waves, Global Model, Local Model, Time-Dependent Scattering.*

ABSTRACT

Recent advances in the development of high-order Absorbing Boundary Conditions (ABC) and Open Boundary Conditions (OBC) for the solution of time-dependent wave problems in unbounded domains will be described. Both types of boundary conditions are imposed on an artificial boundary Γ of a finite computational domain Ω to simulate the behavior of the solution outside Ω. However, an ABC is used in radiation and scattering problems where there are no wave sources outside Ω and all the waves reaching Γ are outgoing, while an OBC is used in fields like numerical weather prediction where both outgoing and incoming waves pass through Γ and should be accounted for accurately.

A formulation of local high-order ABCs recently proposed by Hagstrom and Warburton and based on a modification of the Higdon ABCs, is further developed and extended in a number of ways. First, the ABC is analyzed in new ways and important information is extracted from this analysis. Second, The ABCs are extended to the case of a *dispersive medium*, for which the Klein-Gordon wave equation governs. Third, the case of a *stratified medium* is considered and the way to apply the ABCs to this case is explained. Fourth, the ABCs are extended to take into account *evanescent modes* in the exact solution. The analysis is applied throughout this paper to two-dimensional wave guides. Two numerical algorithms incorporating these ABCs are considered: a standard semi-discrete finite element formulation in space followed by time-stepping, and a high-order finite difference discretization in space and time. Numerical examples are provided to demonstrate the performance of the extended ABCs using these two methods.

In the part of the talk related to OBCs, a two-dimensional global-regional model interaction problem for linear time dependent waves is considered. The setup, which is sometimes called 'one-way nesting,' arises in numerical weather prediction as well as in other fields concerning

waves in very large domains. It involves the interaction of a coarse global model and a fine limited-area (regional) model through an 'open boundary.' The multiscale nature of this general problem is described. The Carpenter scheme, originally proposed in a Note by K.M. Carpenter in 1982 for this type of problem, is then revisited, in the context of the linear scalar wave equation. The original Carpenter scheme is based on the Sommerfeld radiation operator, and thus is associated with low-order accuracy. By replacing the Sommerfeld operator with the high-order Hagstrom-Warburton absorbing operator, a modified Carpenter open boundary condition emerges which possesses high-order accuracy. This boundary condition is incorporated in a computational scheme which uses finite element discretization in space and Newmark time-stepping. Error analysis and numerical tests for wave guides demonstrate the performance of the modified scheme for combinations of incoming and outgoing waves.

The developments to be described here have been reported in the recent publications [1]–[4].

REFERENCES

[1] T. Hagstrom, M. de Castro, D. Givoli and D. Tsemach, "Local High Order Absorbing Boundary Conditions for Time-Dependent Waves in Guides," *J. Computational Acoustics*, **15**, 1–22, 2007.

[2] T. Hagstrom, A. Mar-Or and D. Givoli, "High-Order Local Absorbing Conditions for the Wave Equation: Extensions and Improvements," *J. Computational Physics*, to appear.

[3] A. Mar-Or and D. Givoli, "The Global-Regional Model Interaction Problem — Analysis of Carpenter's Scheme and Related Issues," *Int. J. Multiscale Computational Engineering*, **4**, 617–646, 2006.

[4] A. Mar-Or and D. Givoli, "High Order Global-Regional Model Interaction: Extension of Carpenter's Scheme," *Int. J. Numerical Methods in Engineering*, submitted.