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METRIC-BASED MESH ADAPTATION IN ALE SIMULATION

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ABSTRACT

A challenging problem in numerical simulation of unsteady fluid flows is accurate resolution of sharp solution features including shock waves and contact discontinuities. One of the popular approaches is to move the computational mesh with the solution features. This approach uses a posteriori error estimates to identify regions for mesh adaptation. Many ad hoc techniques have been proposed to regularize the adaptive mesh in the case of highly irregular errors. In article [1], we developed the first mathematical framework for analyzing impact of mesh regularization methods on solution accuracy.

We use the mesh motion method proposed in [2]. Its main advantage is in decoupling of the mesh motion from a numerical method for solving governing PDEs. The price to be paid is necessity of a data transfer (remapping) between meshes. Thus, the total error includes the space discretization error, the remapping error and the time integration error. The analysis of the total error suggests to construct adaptive meshes which are uniform in a metric induced by the solution gradient.

Two practical constraints are usually imposed onto the computational mesh: (1) it should vary smoothly in space and (2) the global variation of the mesh size should be bounded. The first constraint increases accuracy and robustness of time integration methods. The second constraint allows to control the time step. We developed a new adaptive technique for the controlled mesh motion. The technique is based on two metric modifications: smoothing and lifting. The smoothing smears out sharp features in the original metric induced by the gradient of the mesh solution. The lifting prevents the equidistribution principle from blowing out the mesh in regions where both the original and smoothed errors are close to zero. Metric smoothing and lifting are achieved by solving the linear reaction-diffusion equation

$$-(h(x)2v')' + v = u' + c(u'),$$

where u' is the solution derivative, $c(u')$ is the positive lifting constant and $h(x)$ is the piecewise-constant function describing local mesh size and defined on the same mesh as u. The mesh uniform in metric $M = |v|^{0.5}$ meets the aforementioned constraints.

The theoretical analysis of the proposed method results in constructive estimates of the effect of a metric modification on the error. Our software allows us to generate adaptive meshes with desirable properties independently of the problem.

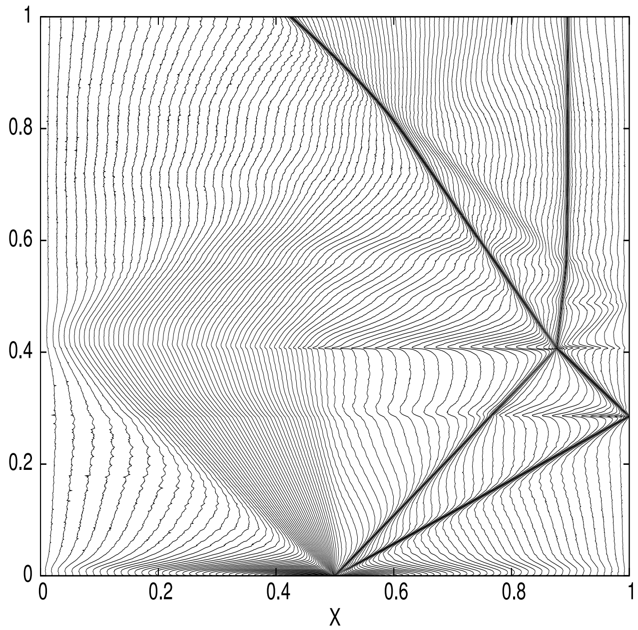

Figure 1: Adaptive solution of the Sod shock tube problem. The 1D tube filled with gas consists of two regions initially separated by a membrane at $x = 0.5$. The gas ($\gamma = 1.4$) to the left of the membrane is more dense ($\rho_L = 1$ and $\rho_R = 0.125$) and is at a higher pressure ($p_L = 1$ and $p_R = 0.1$) compared to the one on the right. The reflecting boundary conditions are imposed at $x = 0$ and $x = 1$. The figure shows space-time trajectories of mesh nodes. Each horizontal cut represents positions of mesh nodes at the chosen time moment. The shock wave (the fastest wave) reflects at the right boundary at $t \approx 0.3$, interacts with the contact discontinuity at $t \approx 0.4$, and remains resolved during the whole simulation. The final solution error is at least twice less compared to that in the pure Lagrangian simulation.

The figure illustrates application of our methodology to the solution of the system of Euler equations. The system describes dynamics of the inviscid, compressible, polytrophic gas in one dimension. The figure shows space-time trajectories of mesh nodes. The trajectories demonstrate propagation and interaction of shock and rerafraction waves with the contact discontinuity. The only user-given parameter was the ratio of maximal to the minimal mesh sizes (20 in this example).

The metric $M(x)$ was based on the gas density derivative. Some wabbling of mesh trajectories in time occurs in regions where the derivative is close to zero and does not affect the solution error: time smoothing of mesh trajectories did not result in error reduction.

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