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# A PARTICLE METHOD FOR STOCHASTIC CONVECTION DIFFUSION EQUATIONS

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## ABSTRACT

We consider the following model for the convection diffusion of a radioactive element in a two dimensional porous media:

$$R\omega(x)\left(\frac{\partial c(x,t)}{\partial t}+\lambda c(x,t)\right)-\nabla\cdot(D(x)\nabla c(x,t))+u(x)\cdot\nabla c(x,t)=f(x,t), \tag{1}$$

with $c \geq 0$ the element concentration, $R > 0$ the latency retardation factor, $\omega > 0$ the effective porosity of the medium, $\lambda > 0$ the half time of the element, $D > 0$ the diffusivity of the medium, $u$ a divergence free velocity field (Darcy velocity) and $f \geq 0$ a source term. The model data $\omega$, $D$ $u$ and $f(x,\cdot)$ are piecewise constant but subjected to uncertainty. They are therefore modeled as random quantities on a abstract probability space $(\Theta, \mathcal{A}, dP)$. In addition to the treatment of the uncertainties, a second difficulty arises from the high Péclet number (ratio of convection and diffusion times) characteristic of the process: discontinuity in the model data and convection dominated transport challenge Eulerian methods (finite volumes, finite elements). To circumvent the latter difficulty, we use a Lagrangian method for the spatial discretization. The random concentration field is discretized using $N$ particles carrying random masses $C_{i=1,\dots,N}$. The particle approximation of the concentration field writes as $c(x,t,\theta)=\sum_{i=1}^{m} C_i(t,\theta)\zeta_\epsilon(x-X_i(t))$, where $\theta \in \Theta$ is a random outcome, $X_i$ is the position of the $i$-th particle and $\zeta_\epsilon$ the smooth kernel of the spatial approximation such that $\lim_{\epsilon\to 0}\zeta_\epsilon(x)=\delta(x)$. At the stochastic level, the discretization makes use of Polynomial Chaos expansions of the random mass $C_i$ and model data. Evolution in time of the particle approximation relies on a splitting approach that treats separately convection, diffusion and decay processes. These treatments involve the displacement of the particles along the mean characteristics of the flow with correction of the random mass $C_i$ to account for the uncertain velocity (convection step), stochastic particles strength exchanges (diffusion step) and an update of the particles mass (decay step). The two former steps follow the methodology recently proposed in [1], while the latter is straightforward. Numerical examples for 2D configurations will be provided to demonstrate the effectiveness of the approach.

## REFERENCES

[1] O.P. Le Maître and O.M. Knio. "A Stochastic particle-mesh scheme for uncertainty propagation in vortical flows". *J. Comput. Phyics*, Vol.**226**, 645–671, 2007.