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HIGH ORDER SCHEMES ON UNSTRUCTURED MESHES FOR TWO-PHASE FREE-SURFACE FLOW WITH STIFF FRICTION

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ABSTRACT

Geophysical phenomena often involve the dynamics of mixtures of materials, for which mathematical models of the multi-field type (either multi-phase or multi-fluid) can be constructed. This presentation centres on numerical methods for multi-field systems for a mixture of two different materials with a free surface under gravity. In particular we shall concentrate our attention to numerical methods for the recently developed two-fluid debris flow model of Pitman and Le [1].

We will present new explicit high order accurate and fully discrete one-step finite volume methods for general non-conservative hyperbolic systems with stiff source terms (e.g. friction) on unstructured triangular meshes. The schemes are based on (i) a WENO reconstruction operator [6] (ii) local time evolution of the data inside each element using a new local space-time discontinuous Galerkin approach that is able to treat stiff source terms robustly and accurately [4] (iii) time interaction of evolved high-order spatial data via the path-conservative schemes recently developed by Parés [5], to deal with non-conservative products, typically present in two-phase flow models.

First, we show that the proposed high order approach is exactly well-balanced, solving a test problem recently proposed by Pelanti et al. [2]. Second, we will solve the one-dimensional Riemann problem proposed and studied extensively by Rhebergen et al. [3] without and with stiff friction source terms. In fact, the numerical results presented here seem to be the first ones that are at the same time very high order accurate in space and time and treat the full model equations of Pitman and Le, also including the stiff friction source terms.

Preliminary results concerning the extension of the approach to 2D on triangular meshes will also be shown.

REFERENCES

[1] E. B. Pitman and L. Le. “A two-fluid model for avalanche and debris flows”, *Philosophical transactions of the Royal Society A*, Vol. **363**, pp. 1573–1601, (2005).

[2] M. Pelanti, F. Bouchut, A. Mangeny and J.P. Vilotte. “Numerical modelling of two-phase gravitational granular flows with bottom topography”, Proceedings of the HYP2006, Lyon, France (2006).

[3] S. Rhebergen, O. Bokhove and J.J.W. van der Vegt. “Discontinuous Galerkin finite element methods for hyperbolic nonconservative partial differential equations”, *Journal of Computational Physics*, Vol. **227**, pp. 1887-1922, (2008).

[4] M. Dumbser, C. Enaux and E. F. Toro. “Finite Volume Schemes of Very High Order of Accuracy for Stiff Hyperbolic Balance Laws”, *Journal of Computational Physics*, Vol. **227**, pp. 3971-4001, (2008).

[5] C. Parés. “Numerical methods for nonconservative hyperbolic systems. A theoretical framewok”, *SIAM J. Num. Anal*, Vol. **44**, pp. 300-321, (2006).

[6] M. Käser, V. A. Titarev and E. F. Toro. “Quadrature-Free Non-Oscillatory Finite Volume Schemes on Unstructured Meshes for Nonlinear Hyperbolic Systems”, *Journal of Computational Physics*, Vol. **226**, pp. 204-243, (2007).