Free Material Optimization: Recent Progress

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ABSTRACT

We present a compact overview of the recent development in free material optimization (FMO), a branch of structural optimization. The goal of FMO is to design the ultimately best material (its mechanical properties and distribution in space) for a given purpose. We show that the current FMO models naturally lead to linear and nonlinear semidefinite programming problems (SDP); their numerical tractability is then guaranteed by recently introduced SDP algorithms.

Free material optimization (FMO) is a branch of structural optimization that gains more and more interest in the recent years. The underlying FMO model was introduced in [2] and later developed in [5] and [1]. The design variable is the full elastic stiffness tensor that can vary from point to point; it should be physically available but is otherwise not restricted. This problem gives the best physically attainable material and can be considered the "ultimate" generalization of the structural optimization problem. The method is supported by powerful optimization and numerical techniques, which allow us to work with bodies of complex initial design and with very fine finite-element meshes, giving thus quite accurate solutions even for bodies with complex geometries.

Recently, FMO has been used for conceptual design of aircraft components; the most prominent example is the design of ribs in the leading edge of the new Airbus A380. In this particular case, the use of the modern optimization techniques leads to a significant weight savings; see Fig. 1.

The goal of this presentation is to show recent developments in FMO formulations and models with the most important technological constraints, such as stress, stability, and displacement constraints. The presentation follows the overview paper [4] and shows the shift in the mathematical programming formulations: the basis of the new models is linear and nonlinear large-scale semidefinite programming; see also [3] for FMO models with stability constraints that again lead to linear or nonlinear SDP problems. This shift in the modeling—from standard nonlinear to semidefinite programming—was enabled by the recent development of new algorithms for (nonlinear) SDP problems. All our FMO models are solved by code PENNON that has been originally developed for this purpose and later turned to a general-purpose NLP and SDP code. In the last section of this presentation we thus give a brief overview of the basic algorithm implemented in PENNON.



Figure 1: Desing of a leading edge rib of A380 using FMO: the wing leading edge; sample FMO result; post-optimization with technological constraints; final product.

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