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MODELS OF SLIGHTLY CURVED DEFECTS IN LAYERED MATERIALS

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ABSTRACT

The defects of layered materials are represented as a small deviation of a boundary or an interface from a flat surface and also slightly curved interfacial cracks and cracks located near an interface. The models of defects are considered within the limits of the 2-D problem of elasticity and may be classified into two groups.

A half plane with a wavy boundary, a curvilinear crack or thin inclusion in a homogeneous plane, two bonded dissimilar half planes with a slightly-undulating interface or a slightly curved interfacial crack are some of models belonging to the first group. The feature of these models is that an analysis of each corresponding problem of continuum mechanics can be applied to layered structures if defect dimensions are far less than minimum distance to a neighbouring boundary or an interface.

In this case, an efficient method of analysis of the elastic problems that leads to the solutions in an explicit form is the boundary perturbation method. As for 2-D problems, this method based on Mushkelishvili approach has been used by Banichuk (1970), Goldstein and Salganik (1970, 1974) and Cotterell and Rice (1980) to study slightly curved cracks and Gao (1991) to analyse nearly circular inclusions and wavy interfaces. Only first-order solutions carried out in different and rather complicated manners have been obtained in these works. In contrast to such solutions, we have developed a unified boundary perturbation technique that gives an algorithm for computing any-order solution of the problems from the first group of models considered. Using Goursat-Kolosov's complex potentials and Mushkelishvili's expressions, it is directly derived formulas for stresses and strains of any-order approximation in an explicit form. This method has been applied to a half-plane with a slightly curved boundary [1] and other defects from the first group of models. The method and an analysis of the first-order perturbation solution are presented in the paper for a slightly curved interface and a slightly curved interfacial crack and a dependence of stress concentration and stress intensity factors upon a shape of defects and elastic parameters of a bimaterial is discussed.

For the models of the second group, the boundary perturbation method is used together with so-called combined method that leads to Fredholm integral equation of the second kind. First,

the combined method has been applied to the problems of a rectilinear crack located in a half plane [2, 3] and in a strip [4]. The foundation of the last method is the superposition principle and the optimum combination of step-by-step method of solving the boundary integral equation derived and direct solution of this equation via a conventional technique.

A slightly curved crack located near the interface in dissimilar materials is one of defects of the second group. We solve the 2-D elastic problem for such a defect as a sum of two problems: a) a slightly curved crack in a homogeneous plane with the action of an unknown traction on the crack surfaces and b) two connected dissimilar half planes under remote loading with jumps of a traction and displacements at the interface expressed in terms of the traction of the first problem. Supposing that maximum deviation ε of the crack from a reference straight one is much smaller than the length of the reference crack, we expand the unknown traction as a power series in ε. Based on Goursat-Kolosov's complex potentials that are expanded as a power series in ε as well, the solution of the problem is reduced to a successive solution of Fredholm integral equations of the second kind for finding the expansion complex coefficients of each order approximation. Integral equations differ only in right hand members depending on all solutions of previous levels of approximation. In order to obtain an approximate solution of an integral equation, we use a polynomial approximation of the unknown expansion coefficients that are functions of a point at the surface of the reference crack. Finely, the integral equation transforms into a system of algebraic equations for computing polynomial coefficients. It is carried out an algorithm that one can use to find any-order perturbation solution. Based on the first-order solution, a dependence of stress intensity factors and stresses at the interface on parameters of the problem is demonstrated in the paper for a crack the shape of which is determined by a power function.

Perturbation technique presented in the paper can be applied to different models of defects of the second group such as a slightly curved crack in a layer, a wavy interface between a thin layer and a substrate and others.

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