8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

High–Order/hp–Adaptive Discontinuous Galerkin Finite Element Methods for Compressible Fluid Flows

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Key Words: *Discontinuous Galerkin methods, a posteriori error estimation, hp–adaptivity, compressible fluid flows.*

ABSTRACT

We present an overview of some recent developments concerning the *a posteriori* error analysis of h– and hp–version finite element approximations to compressible fluid flows. After highlighting some of the conceptual difficulties in error control for problems of hyperbolic/nearly–hyperbolic type, such as the lack of correlation between the local error and the local finite element residual, we concentrate on a specific discretisation scheme: the hp–version of the discontinuous Galerkin finite element method. This method is capable of exploiting both local polynomial–degree–variation (p–refinement) and local mesh subdivision (h–refinement), thereby offering greater flexibility and efficiency than numerical techniques which only incorporate h–refinement or p–refinement in isolation.

We shall be particularly concerned with the derivation of *a posteriori* bounds on the error in certain output functionals of the solution of practical interest; relevant examples include the lift and drag coefficients for a body immersed into a fluid, the local mean value of the field or its flux through the outflow boundary of the computational domain, and the pointwise evaluation of a component of the solution.

By employing a duality argument we derive so–called weighted or Type I *a posteriori* estimates which bound the error between the true value of the prescribed functional, and the actual computed value. In these error estimates, the element residuals of the computed numerical solution are multiplied by local weights involving the solution of a certain *dual* or *adjoint* problem. On the basis of the resulting *a posteriori* error bound, we design and implement an adaptive finite element algorithm to ensure reliable and efficient control of the error in the computed functional with respect to a user–defined tolerance. For the hp–version of the discontinuous Galerkin finite element method, the decision as to whether to h–refine or p–refine an element is based on estimating the local analyticity of the primal and dual solutions via truncated Legendre series expansions. The performance of the resulting hp–refinement algorithm is demonstrated through a series of numerical experiments.

This research has been funded by the EU under the ADIGMA project.

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