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VERIFICATION TEST OF MIXED HIGH-ORDER NUMERICAL CODES FOR LAMINAR-TURBULENT TRANSITION SIMULATION

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ABSTRACT

A verification test using the Method of Manufactured Solutions (MMS) was performed on a Direct Numerical Simulation code of mixed high-order of accuracy. This test is based on the formulation of an exact solution for the Navier-Stokes equations modified by the addition of a source term (Roache, 1998; Roy, 2005). In the test, a mesh refinement test gave the order of accuracy of the calculations. The results showed that for code with mixed high-order of accuracy, much care must be exercised both in choosing the manufactured solutions and in interpreting the results.

In general, simulations of laminar-turbulent transition, turbulence and aeroacustical flows require the use of numerical methods of high-order of accuracy. This is necessary because the simulation of small spatial and temporal scales is of fundamental importance to the interpretation of the physical problems. When nonperiodic boundary conditions are implemented, such as wall bounded flow, the use of mixed high-order approximation is required because near and at boundary, an approximation relatively at lower order of accuracy is needed to ensure numerical stability and obtain optimized algorithms.

The current numerical code is aimed at simulating the temporal evolution of instability waves in a plane Poiseuille flow (Silva et al. , 2007). The governing equations were solved in a vorticity-velocity formulation for a two-dimensional incompressible flow.

In the current test, particular attention was paid to both the boundary conditions of the physical problem of interest and the nonlinear calculations. Therefore an exact solution was manufactured that was nonlinear and satisfied the boundary conditions of the physical problem. It was possible to manufacture an exact solution that imitated an Tollmien-Schlichting instability wave in a nonlinear stage:

$$u(x,y) = A\mathrm{e}^{y}y(y+1-\sqrt{5})(y+1+\sqrt{5})(y-2)\cos(\alpha x), \tag{1}$$

$$v(x,y) = A\sin(\alpha x)\alpha\mathrm{e}^{y}y^{2}(y^{2}-4y+4), \tag{2}$$

$$\omega(x,y) = -A\mathrm{e}^{y}\cos(\alpha x)(8y^{2}-8+4\alpha^{2}y^{2}+\alpha^{2}y^{4}-4\alpha^{2}y^{3}-4y^{3}+8y-y^{4}). \tag{3}$$

Figure 1 shows the behavior of the spatial average error (indicated by E_m) for the vorticity. The mesh refinement was performed simultaneously in the x and $y-$directions using six different meshes ($m = 1,\ldots,6$). The number of intervals in that directions doubled from one mesh to the following mesh.

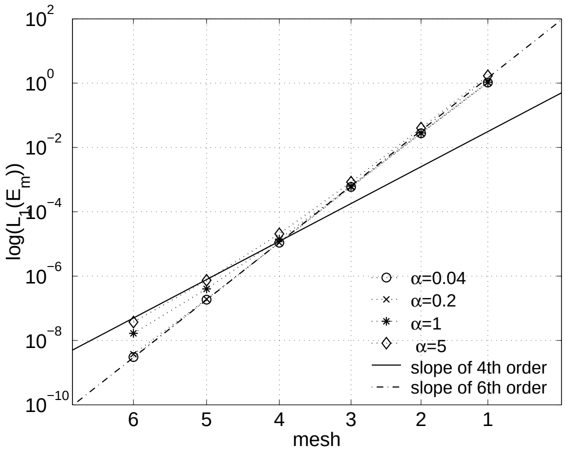

Figure 1: Behavior of E_m for the vorticity at four different values of α. The results were showed in a semi-logarithmic scale.

The results showed that, for high-order codes, it can be very difficult to reach the asymptotic range of accuracy for high-order codes. In fact, by using sufficiently refined meshes, the numerical error was reduced to the level of the round off error but, the asymptotic range was not reached. However, by judiciously modifying the value of α in the manufactured solution, the 4^{th} order of accuracy asymptotic range was reached. Note that the width of the asymptotic range also depended on the test case. Furthermore, for the results that were outside the asymptotic range, the observed order was, nevertheless, found to be consistent with the other discretization order of accuracy employed in the code.

Another interesting fact indicated by the results was that, depending on the region of the domain and on the variable, the numerical error can be very different in magnitude. This was indicated by the analysis of the local discretization error for the vorticity at different points at the domain. The results indicated a predominant numerical error for the calculation of the vorticity at the wall. This occurred because in the vorticity-velocity formulation it is necessary that the boundary condition for the vorticity be calculated rather than specified directly. This calculation used approximation less accurate that those used at other regions of the domain. Nevertheless little contamination from the predominant error of the vorticity calculation at the wall was observed in other regions.

The results showed that the MMS can be used to test mixed high-order of accuracy numerical codes if the results of the test are produced and interpreted with care.

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