8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

THE MULTISCALE ANALYSIS FOR INCOMPRESSIBLE FLOW OF MAXWELL FLUID

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Key Words: *Non-Newtonian Fluid, Multicale, Homogenization, Finite Volume Method.*

ABSTRACT

It is well known that multiscale phenomena are pervasive in physics and engineering. However, it is almost impossible to capture all small scale features by direct numerical simulations using the standard finite element method or finite difference method if very fine meshes are used. Therefore, it is necessary to develop an effective coarse grid method, which can capture the information both large scale and the influence of small scale on large scale but does not require obtaining directly all small scale physical features, so that some problems related with fine property can be solved.

The purpose of this article is to develop a method of multiscale analysis for non-Newtonian fluid which is rare in the research of multiscale analysis. We start our studies with a kind of linear viscoelastic non-Newtonian fluid, namely, the following incompressible flow of Maxwell fluid with highly oscillating initial data

$$\begin{aligned}
&\nabla \cdot \mathbf{u}^{\varepsilon} = 0 \\
&\partial_t \mathbf{u}^{\varepsilon} + (\mathbf{u}^{\varepsilon} \cdot \nabla)\mathbf{u}^{\varepsilon} = -\nabla p^{\varepsilon} + \nabla \cdot \mathbf{g}^{\varepsilon} \\
&\lambda \partial_t \mathbf{g}^{\varepsilon} + \mathbf{g}^{\varepsilon} = \mu(\nabla \mathbf{u}^{\varepsilon} + (\nabla \mathbf{u}^{\varepsilon})^T) \\
&\mathbf{u}^{\varepsilon}(0,\mathbf{x}) = \mathbf{U}(\mathbf{x}) + \mathbf{W}(\mathbf{x}, \frac{\mathbf{x}}{\varepsilon}) \\
&\mathbf{g}^{\varepsilon}(0,\mathbf{x}) = \mu(\nabla \mathbf{u}^{\varepsilon}(0,\mathbf{x}) + (\nabla \mathbf{u}^{\varepsilon}(0,\mathbf{x}))^T)
\end{aligned}$$

where $\mathbf{u}^{\varepsilon}(t,\mathbf{x})$, $p^{\varepsilon}(t,\mathbf{x})$, and $\mathbf{g}^{\varepsilon}(t,\mathbf{x})$ are the velocity vector, pressure field and stress tensor respectively. For the initial velocity field, $\mathbf{U}(\mathbf{x})$ is assumed to a given smooth mean velocity field in $\boldsymbol{R}^2$, and $\mathbf{W}(\mathbf{x},\mathbf{y})$ with $\mathbf{y} = \mathbf{x}/\varepsilon$ is the oscillatory component of velocity field which is a smooth function of $\mathbf{x}$ and $\mathbf{y}$, and is periodic in $\mathbf{y}$. Moreover, the function $\mathbf{W}$ is assumed to have zero mean

$$\langle \mathbf{W} \rangle = \int_{\Omega} \mathbf{W}(\mathbf{x},\mathbf{y}) d\mathbf{y} = 0$$

where $\Omega = [0,1]\times[0,1]$. These conditions are typical assumptions in homogenization theory. That means the initial condition for velocity field contains the information both large scale and the small scale.

In comparison with the incompressible flow of Newtonian fluid, the above governing equations of non-Newtonian fluid are more complex due to the existence of constitutive equations. The influence and the evolution of the stress field must be considered in developing the method of multiscale analysis for the incompressible flow of Maxwell fluid.

By introducing the Lagrangian map and making the appropriate multiscale asymptotic expansions for the velocity, pressure and stress field, a well-posed homogenized system including the homogenized equations and the cell problem is obtained for the incompressible flow of Maxwell fluid. And then, the homogenized system and the original governing equation are solved numerically by using the finite volume method based on momentum interpolation idea in the collocated grids. The comparisons between the results of direct numerical simulations and the multiscale solutions indicate that

(1) the multiscale model can not only capture the large scale features of the incompressible flow of Maxwell fluid, but also show the interactions between the large scale and small scale. Therefore, our multiscale analysis is effective and feasible.

(2) The finite volume method based on momentum interpolation idea in the collocated grids can avoid the decoupling both velocity-pressure and velocity-stress, so that the homogenized system and the original governing equation are solved accurately.

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