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# DISCRETE LIMIT ANALYSIS FOR GRW BY USING HPM WITH PENALTY DENSITY FUNCTION

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## ABSTRACT

## 1. INTRUDUCTION

This paper presents the new approach for the modelling of Geotextile Reinforced soil Wall(GRW) by using Hybrid-type Penalty Method(HPM)[1] with penalty density function. In HPM, it assumes the linear displacement field with rigid displacement, rigid rotation and constant strain in each element. So, it can be deal with the fracture on the intersection boundary and yielding in each element [2]. In the modelling of GRW, it used the penalty density function. In this study, we will show the formulation of penalty density function and examine the example which comparing with FEM and experiment.

## 2. HYBRID-TYPE PENALTY METHOD WITH PENALTY DENSITY FUNCTION

Present method (HPM:Hybrid-type Penalty Method) introduce hybrid displacement model based on the modified principle of virtual work. In this model, subsidiary conditions are introduced into the framework of the variational expression with Lagrange multipliers. Physical meaning of the Lagrange multiplier is equal to the surface force on the intersection boundary. Then, the concept of the spring of RBSM [3] is applied to the Lagrange multiplier. Compatibility of the displacement on the intersection boundary is approximately introduced using the penalty as a spring constant. Therefore, the displacement filed can be assumed for each element without restraining by the condition of compatibility. So, we assume the linear displacement field $\boldsymbol{u}^{(e)}$ with rigid displacement and rotation $\boldsymbol{d}^{(e)}$ and constant strain $\varepsilon^{(e)}$ in each sub-domain ($e$) as follows:

$$\boldsymbol{u}^{(e)} = \boldsymbol{N}_d{}^{(e)}\boldsymbol{d}^{(e)} + \boldsymbol{N}_\varepsilon{}^{(e)}\boldsymbol{\varepsilon}^{(e)} \tag{1}$$

Applying Lagrange multipliers $\boldsymbol{\lambda}$ to the idea of RBSM, the surface force $\boldsymbol{\lambda}_{<ab>}$ of boundary between sub-domain $\Omega^{(a)}$ and $\Omega^{(b)}$ is able to express as follow:

$$\boldsymbol{\lambda}_{<ab>} = \boldsymbol{k}\cdot\boldsymbol{\delta}_{<ab>} \quad , \quad \boldsymbol{k} = \boldsymbol{p} \tag{2}$$

where $\boldsymbol{k}$ is the spring constant matrix, $\boldsymbol{p}$ is a matrix with penalty function.

Substituting (1), (2) into the eqution of hybrid type virtual work, it can be obtained as following final conclusion of the discretized governing equation for whole domain.

$$\boldsymbol{KU} = \boldsymbol{P} \quad , \quad \boldsymbol{K} = \sum_{e=1}^{M}\boldsymbol{K}^{(e)} + \sum_{s=1}^{N}\boldsymbol{K}_{<s>} \quad , \quad \boldsymbol{P} = \sum_{e=1}^{M}\boldsymbol{P}^{(e)} \tag{3}$$

In the case of treating the geogrid, we need to consider the pelnalty both upper and lower side of Grogrid. We propose the penalty density funcion $\beta$ to avoid concentraion of penalty.

$$\beta = \frac{1}{h_1 + h_2} \quad , \quad k_n = k_t = \frac{P}{h_1 + h_2} = \beta \cdot P = P' \qquad (4)$$

where, $h_1$ and $h_2$ is the pendicular line from center of figure to the element boundary.

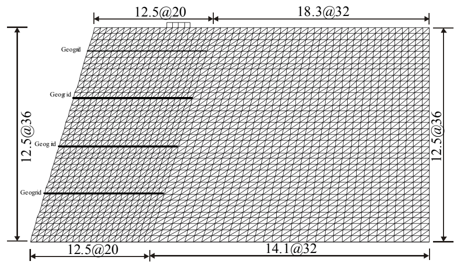


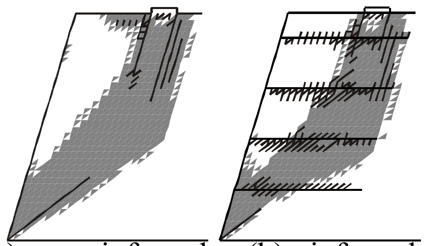

Fig.1 Analysis model

## 3. NUMERICAL EXAMPLE

Fig.1 shows the numerical model and mesh division. It's analyzed under the condition of plane strain and Mohr-Coulomb. The material properties are shown in Table 1. As a result, Fig.2 shows the slip line and yielding area (a:non-reinforced, b:reinforced) that considered the fracture on the intersection boundary and yielding in each element at the same time. Comparing with (a) and (b), yielding area shows the same fractual pattern. In the case of (b), geogrid prevent the slip line. And it can be keeping stability of the ground. And Fig.3 shows the Load-Displacement curve. As for HPM, initial stiffness accorded with FEM and experiment value. And we obtaine the similer curve after plasticity.

Table.1

| Ground | |
|---|---|
| Young's moduls(kN/m$^2$) | 2,500 |
| Poison's ratio | 0.3 |
| Internal friction | 35 |
| Cohesion's force(kN/m$^3$) | 1.8 |
| GRW | |
| Young's moduls(kN/m$^2$) | 1.6E+6 |
| Cross section(m$^2$/m) | 1.0E+4 |



(a)non-reinforced (b)reinforced

Fig.2 Slip line(28.5kN/m$^2$)

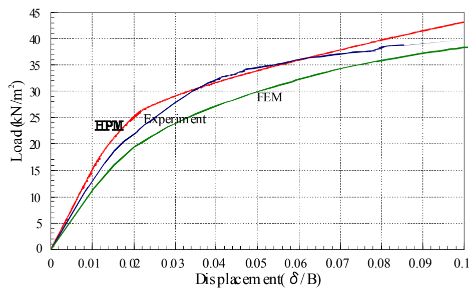



Fig.3 Load-Displacement curve

## 4. CONCLUSION

In this study, it's shown the modeling of the GRW by using penalty density function. And it examines the effect of reinforcement by numerical analysis. Proposed method is shown by comparing with experiment results and numerical results of FEM. As a result of comparing about the Load-Displacement and slip line curve, it's obtained the similar solution with experiment. And present method can be deal with the discontinuous progressive fracture. Namely, it is shown that this method is the usefulness to the modeling of Reinforced Soil.

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