# AUTOMATIC TETRAHEDRAL MESH GENERATION FOR ENVIRONMENTAL PROBLEMS 

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#### Abstract

In this paper we present new ideas and applications to environmental problems of an innovative tetrahedral mesh generator which was introduced in [1,2]. A local refinement/derefinement algorithm for nested triangulations [3] and a simultaneous untangling and smoothing procedure [4] are the main involved techniques. The mesh generator can be applied to 3-D complex domains whose boundaries are projectable on external faces of a high quality and coarse hexahedral mesh of an object meccano. The domain surfaces must be given by a mapping between meccano surfaces and object boundary. This mapping can be defined by analytical or discrete functions. At present we have fixed mappings with orthogonal, cylindrical and radial projections, but other one-to-one projections may be considered. The mesh generator starts from a coarse tetrahedral mesh which is automatically obtained by the subdivision of each hexahedra into six tetrahedra. The main idea is to construct a sequence of nested meshes by refining only those tetrahedra which have a face on the meccano boundary. The virtual projection of meccano external faces defines a valid triangulation on the domain boundary. Then a 3-D local refinement/derefinement is carried out such that the approximation of domain boundary surfaces verifies a given precision. Once this objective is reached, those nodes placed on the meccano boundary are really projected on their corresponding true boundary, and inner nodes are relocated using a suitable mapping. As the mesh topology is kept during node movement, poor quality or even inverted tetrahedra could appear in the resulting mesh. For this reason, we finally apply the simultaneous untangling and smoothing procedure [4], which is based on local node relocation technique. Besides, a smoothing of the boundary surface triangulation could be applied before the movement of inner nodes of the domain by using the procedure presented in [5]. Specifically, in this paper we apply our technique to automatically construct 3-D triangulations of domains oriented to solve environmental problems. We will show test examples which justify the minimal user intervention and the efficiency of the procedure. As example, the interior faces of a meccano (formed by six cuboid pieces) are projected on the surface of the Earth and the exterior ones are projected on an spherical surface included in the Atmosphere, see Figure 1. In this example, we have


automatically constructed an adaptive discretization to approach the height of Earth surface with a given precision. To define the topography we have used the global digital elevation model GTOPO30 (http://edc.usgs.gov/products/elevation/gtopo30/gtopo30.html).


Figure 1: Resulting adaptive tetrahedral mesh of a section of an atmosphere shell.
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