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SHAPE OPTIMIZATION FOR ELLIPTIC PDE: OPTIMALITY CONDITIONS, CONVERGENCE AND STABILITY

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ABSTRACT

In this talk we consider the numerical solution of shape optimization problems. In particular, shape optimization is often used as contour refinement step, for instance after finding the rough design structure of an workpiece by topology optimization methods, see Bendsoe and Sigmund [1], Lewinski and Sokolowski [7], Sokolowski and Zochowski [8].

Based on a first and second order shape calculus (see e.g., Sokolowski and Zolesio [9], Delfour and Zolesio [2]), we investigate first and second order optimality conditions. We will explain the two-norm discrepancy that arises from the mapping properties of the shape Hessian. The shape Hessian is a pseudodifferential operator which acts on the shape variation. It is a continuous bilinear form not only in the space arising from shape calculus but also with respect to an essentially weaker norm. Strict coercivity in the weaker space is needed to ensure local optimality, provided that a refined second order remainder estimate is satisfied.

Based on the coercivity of the shape Hessian, we can classify elliptic shape optimization problems as being either well-posed or ill-posed problems, $[3-5]$. Moreover, we present a concept for proving existence and convergence of approximate shapes in case of well-posedness, [6]. Especially this comprises the *order of convergence* of the shape approximation.

In the second part of the talk, we discuss the efficient numerical solution of elliptic shape optimization problems. In particular, the use of the Hadamard representation of the shape gradient will avoid computation of either shape sensitivities of the state or/and so called mesh sensitivities, both known as beeing computational expensive. For the numerical solution of the state equation by boundary element methods, we present a wavelet Galerkin method of optimal complexity. It is shown that combination of boundary element methods and boundary integral representations of shape derivatives may result in an algorithm that works completely on the boundary. That is, no further information is needed from inside

the domain. Furthermore, we sketch the coupling of finite element and boundary element methods for other objectives like the L_2-tracking of the state on a compact, but fixed subregion of the unknown domain.

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