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Error estimation and adjoint-based refinement for multiple force coefficients in aerodynamic flow simulations

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ABSTRACT

In this talk we give an overview of recent developments on adaptive higher order Discontinuous Galerkin discretizations for the use in computational aerodynamics at the DLR in Braunschweig. In particular, this includes some of the most recent developments and results achieved in the EU project ADIGMA.

Important quantities of interest in aerodynamic flow simulations are the aerodynamic force coefficients including the pressure induced and the viscous stress induced drag, lift and moment coefficients, respectively. *A posteriori* error estimation and goal-oriented (adjoint-based) refinement approaches have been developed for the accurate and efficient computation of *single* target quantities. These approaches are based on computing an adjoint solution related to each of the specific target quantities under consideration. The resulting goal-oriented adaptively refined meshes are specifically tailored to the accurate computation of the target quantity under consideration.

This approach has been extended to the accurate and efficient computation of *multiple* target quantities. Instead of computing multiple adjoint solutions, one for each target functional, the new approach is based on the computation of one adjoint and one adjoint adjoint solution. This way only two auxiliary problems are required irrespective of the number on target functionals. This technique has first been developed for the inviscid Burgers equation and applied to multiple point values in [8]. This approach has now been extended to the error estimation and goal-oriented refinement for multiple aerodynamic force coefficients, see [4]. The practical performance of this approach is demonstrated for a 2d laminar compressible flow.

Provided the adjoint solution related to a target functionals is sufficiently smooth the corresponding error representation can be bounded from above by an error estimate which includes the primal residuals but is independent of the adjoint solution. By localizing this error estimate so-called *residual-based* indicators can be derived. Mesh refinement based on these indicators leads to meshes which resolve *all* flow features irrespective of any specific target quantity. The residual-based indicators have been derived

and implemented for 3d laminar flows. The performance of these indicators will be demonstrated for a laminar 3d ADIGMA test case.

Up to now *a posteriori* error estimation and goal-oriented refinement approaches in aerodynamics have been applied mainly to two-dimensional flows. We give first results extending and applying this approach to three-dimensional flows. In particular, for a laminar 3d ADIGMA test case we show the accuracy of the error estimation with respect to aerodynamic force coefficients. Furthermore, we demonstrate the performance of the adjoint-based refinement approach for the accurate and efficient computation of these coefficients in comparison to the residual-based refinement approach. Finally, these refinement approaches can combined with anisotropic refinement as described in [10].

We note, that the presented results are based on an optimal order interior penalty Discontinuous Galerkin method [9]. In particular, the discretization (including boundary conditions) is consistent and adjoint consistent and the target functionals (aerodynamic force coefficients) are evaluated in an adjoint consistent formulation, see [3,5,7]. The corresponding discrete adjoint problem is a consistent discretization of the continuous adjoint problem. The discrete adjoint solution corresponding to an adjoint consistent discretization is known to be smooth.

All computations have been performed using the DG flow solver `PADGE` [6] which is based on the `deal.II` library [1,2].

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